Projeto e Análise de Algoritmos

Aula 12 – Ordenação Topológica

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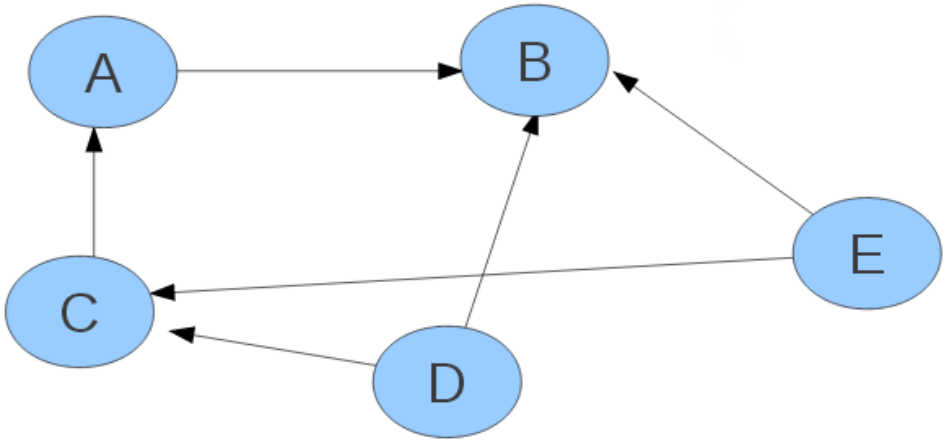
Problema

- Um conjunto de N tarefas precisam ser executadas.
- Tarefas são dependentes:
  - Exemplo: tarefa B só pode ser executada depois de A;
  - Podemos modelar o problema como um grafo direcionado.
- Problema: **Qual ordem de execução não viola as dependências?**

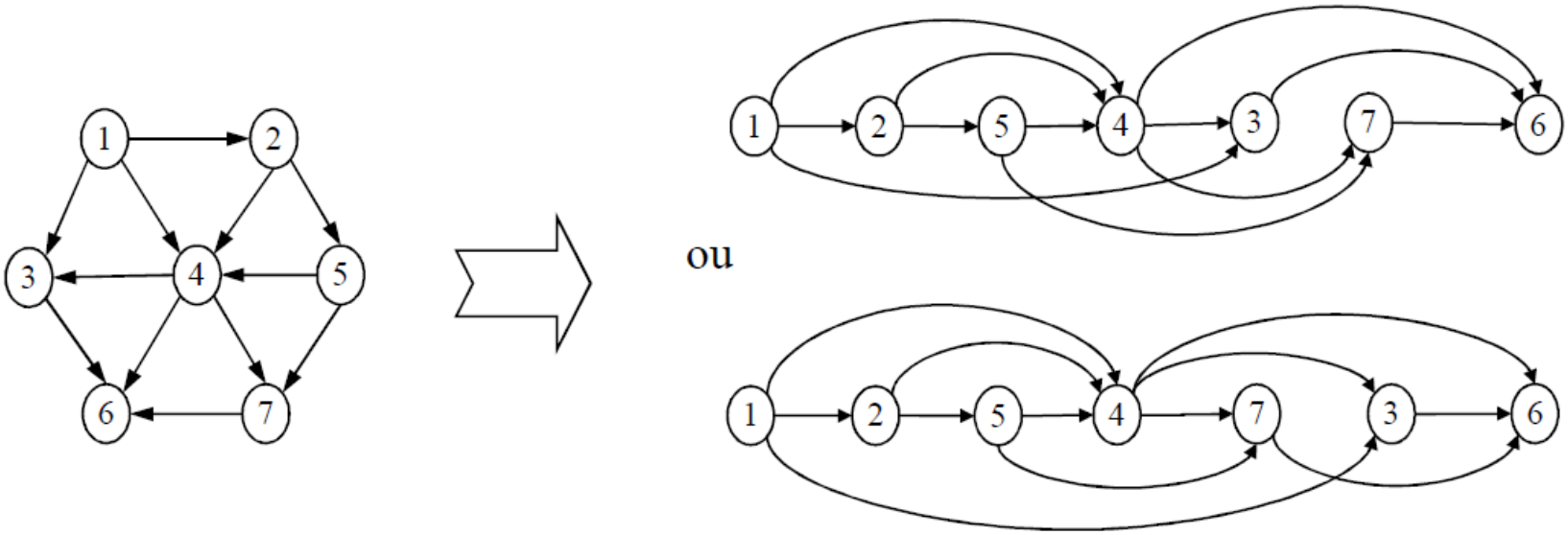
Problema – Exemplo

- **Exemplo:**
  - B depende de A
  - A depende de C
  - C depende de D
  - B depende de E
  - B depende de D
  - C depende de E



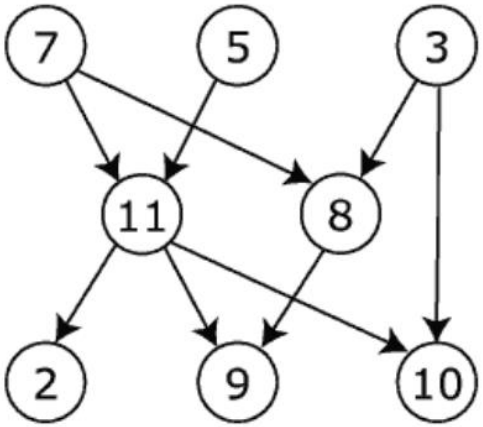
Ordenação Topológica

- Ordenação linear dos vértices de um DAG (Grafo Acíclico Dirigido) tal que, se existe uma aresta (*v*, *w*) no grafo, então *v* aparece antes de *w*.
  - Impossível se o grafo for cíclico;
  - Não é necessariamente única.



Ordenação Topológica

- O grafo abaixo tem diversas ordenações topológicas possiveis:



- 7, 5, 3, 11, 8, 2, 9, 10
- 3, 5, 7, 8, 11, 2, 9, 10
- 3, 7, 8, 5, 11, 10, 2, 9
- 5, 7, 3, 8, 11, 10, 9, 2
- 7, 5, 11, 3, 10, 8, 9, 2
- 7, 5, 11, 2, 3, 8, 9, 10

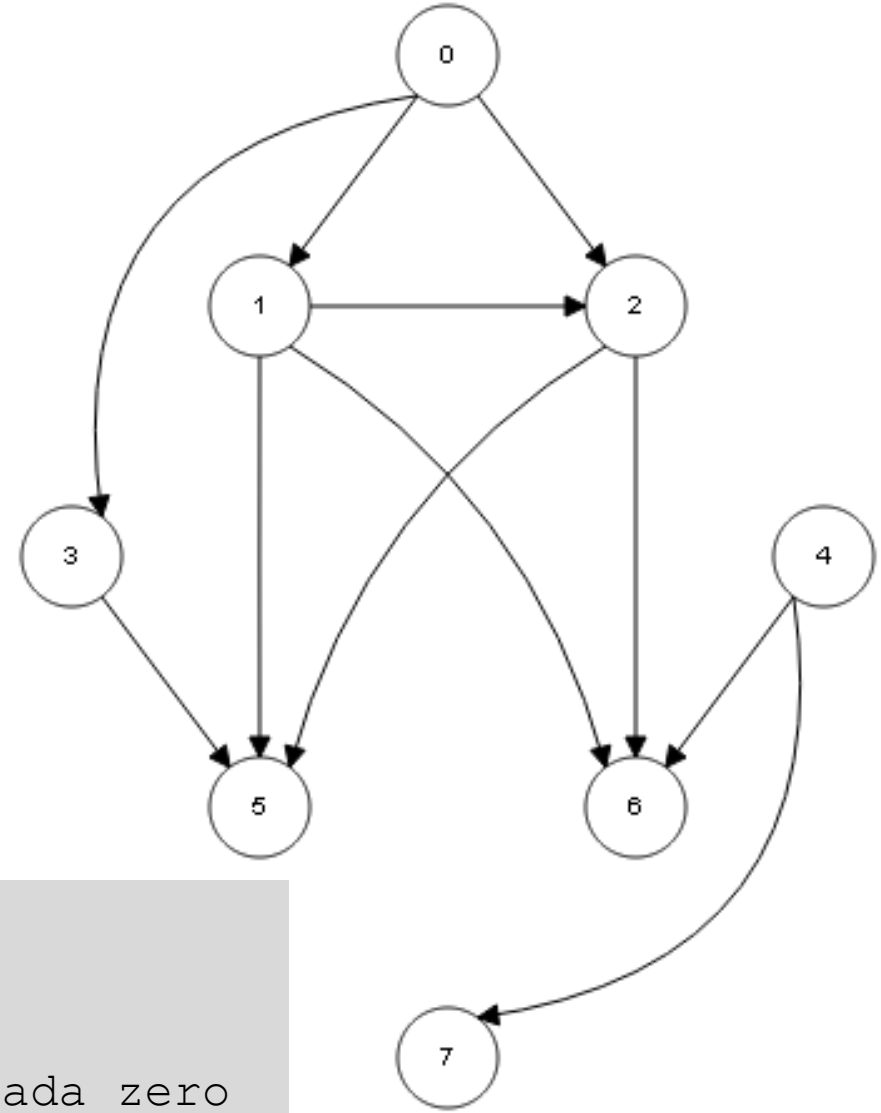
- Algoritmos:
  - Eliminação de vértices (algoritmo de Kahn);
  - Utilizando uma busca em profundidade;

Algoritmo de Kahn

```
OrdenacaoTopologicaKahn(G)
  L ← ∅
  for each uv ∈ A[G]
    I[v] ← I[v] + 1
  S ← vertices com grau de entrada zero (S = pilha)
  while S ≠ ∅
    v ← unstack(S)
    stack(v, L)
    for each u ∈ Adj[v]
      I[u] ← I[u] - 1
      if I[u] = 0
        stack(u, S)
  return L
```

Algoritmo de Kahn

	I	S	L
0	0	/	/
1	0		
2	0		
3	0		
4	0		
5	0		
6	0		
7	0		

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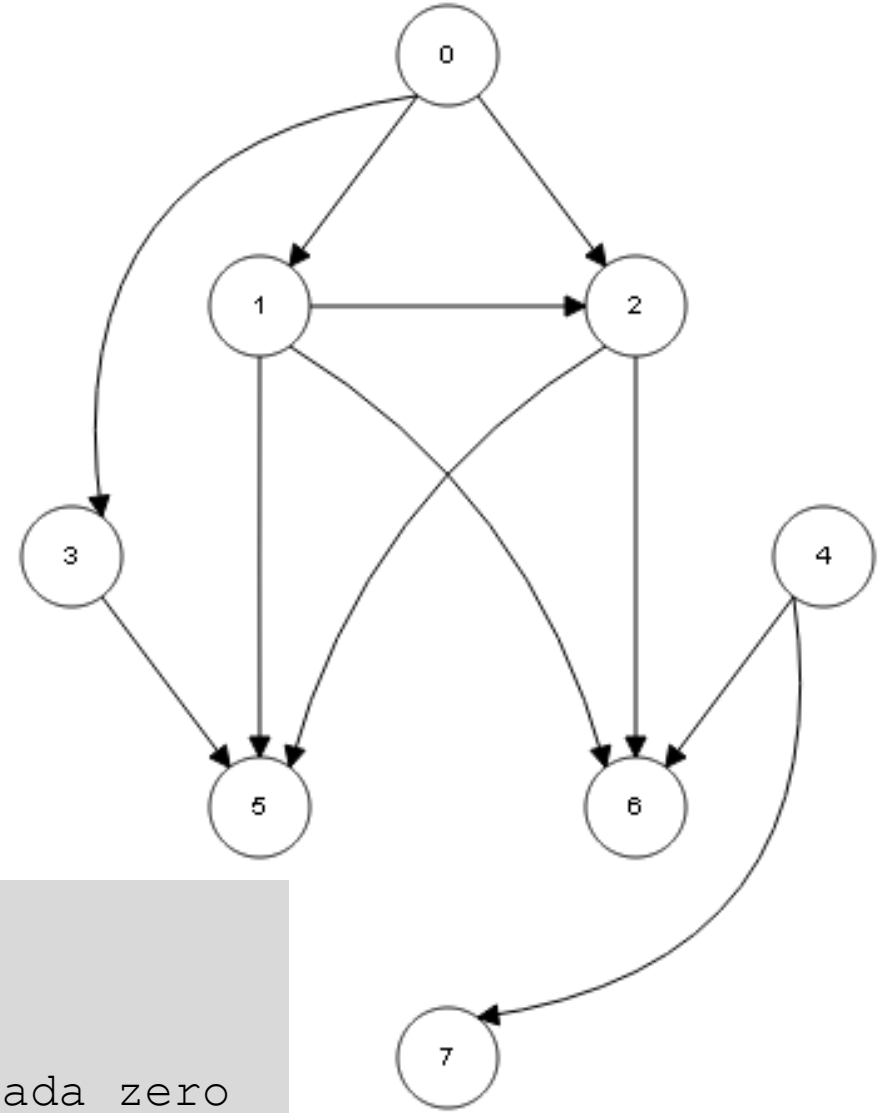
L ← ∅
for each uv ∈ A[G]
  I[v] ← I[v] + 1
S ← vertices com grau de entrada zero
```

Algoritmo de Kahn

	I	S	L
0	0	/	/
1	1		
2	2		
3	1		
4	0		
5	3		
6	3		
7	1		



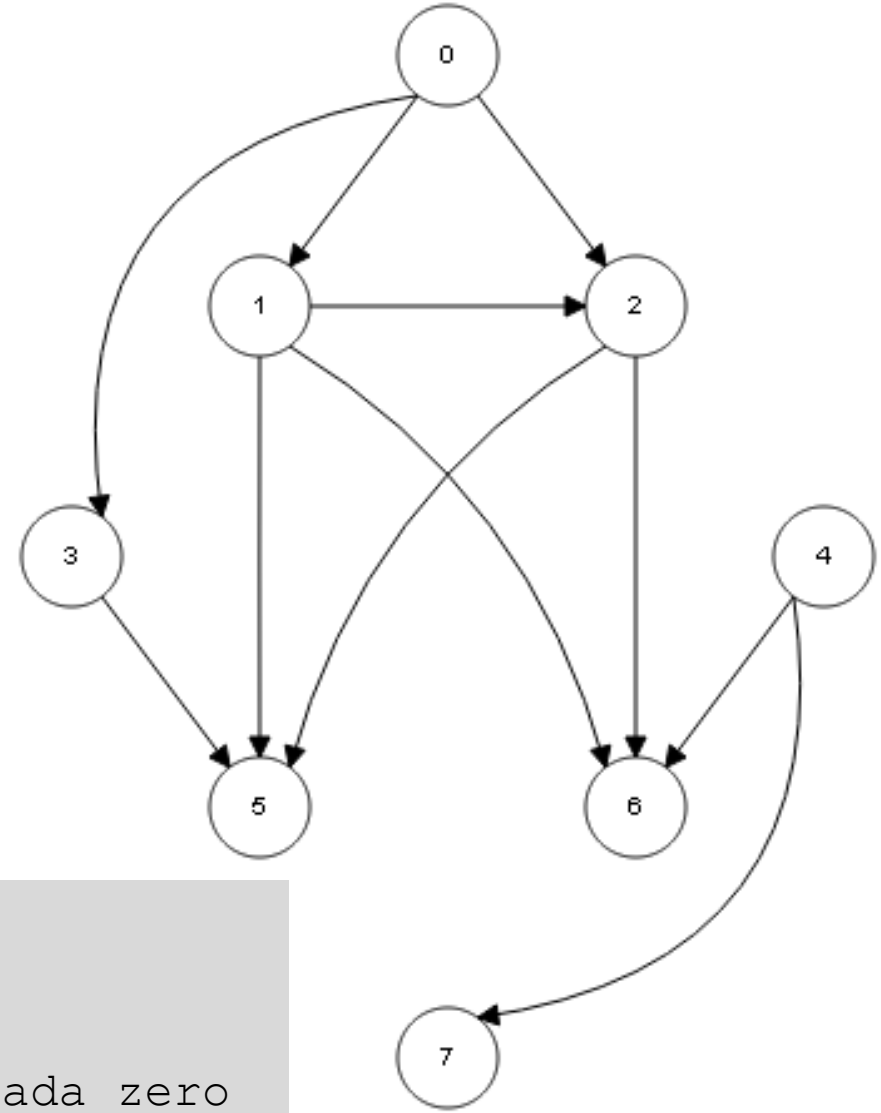

```
L ← ∅
for each uv ∈ A[G]
  I[v] ← I[v] + 1
S ← vertices com grau de entrada zero
```

Algoritmo de Kahn

	I
0	0
1	1
2	2
3	1
4	0
5	3
6	3
7	1

S
0
4

L
/

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L ← ∅
for each uv ∈ A[G]
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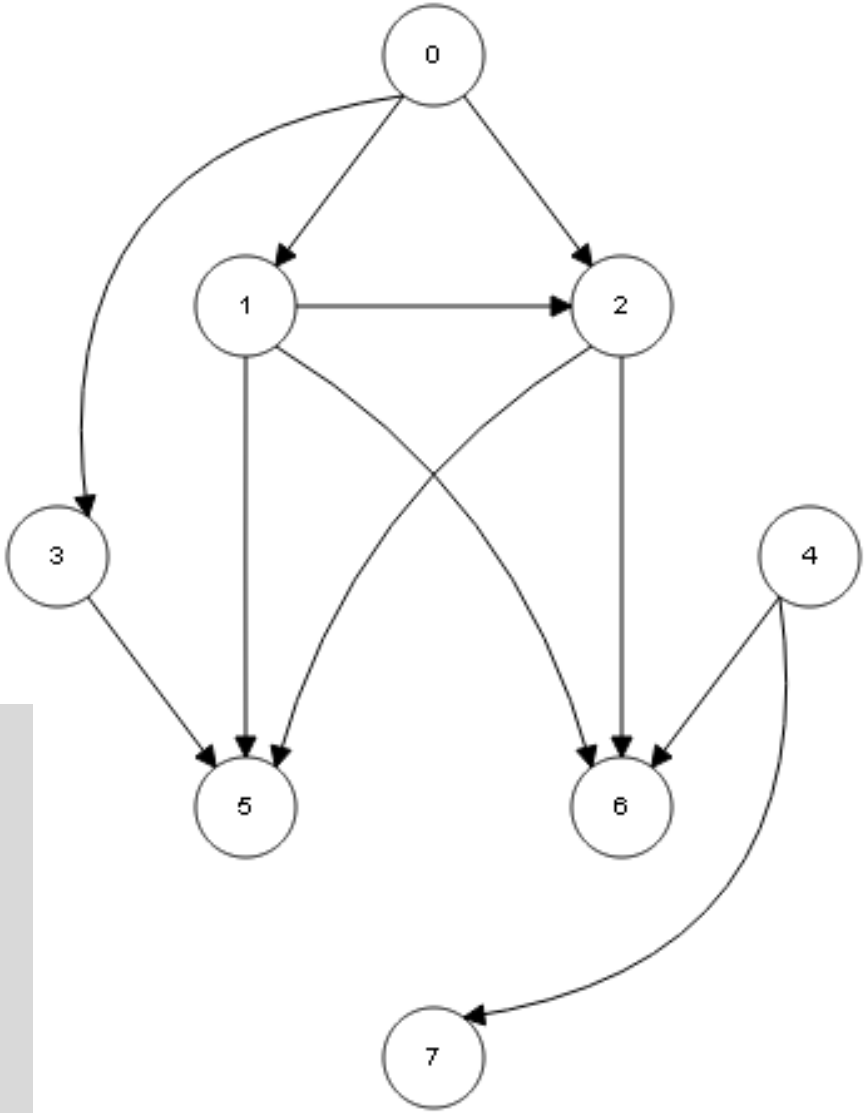
```

Algoritmo de Kahn

	I	S	L
0	0	0	4
1	1		
2	2		
3	1		
4	0		
5	3		
6	3		
7	1		

```
while S ≠ ∅
  v ← unstack(S)
  stack(v, L)
  for each u ∈ Adj[v]
    I[u] ← I[u] - 1
    if I[u] = 0
      stack(u, S)
```



Algoritmo de Kahn

	I	S	L
0	0	0	4
1	1		
2	2		
3	1		
4	0		
5	3		
6	3		
7	1		

```
while S ≠ ∅
  v ← unstack(S)
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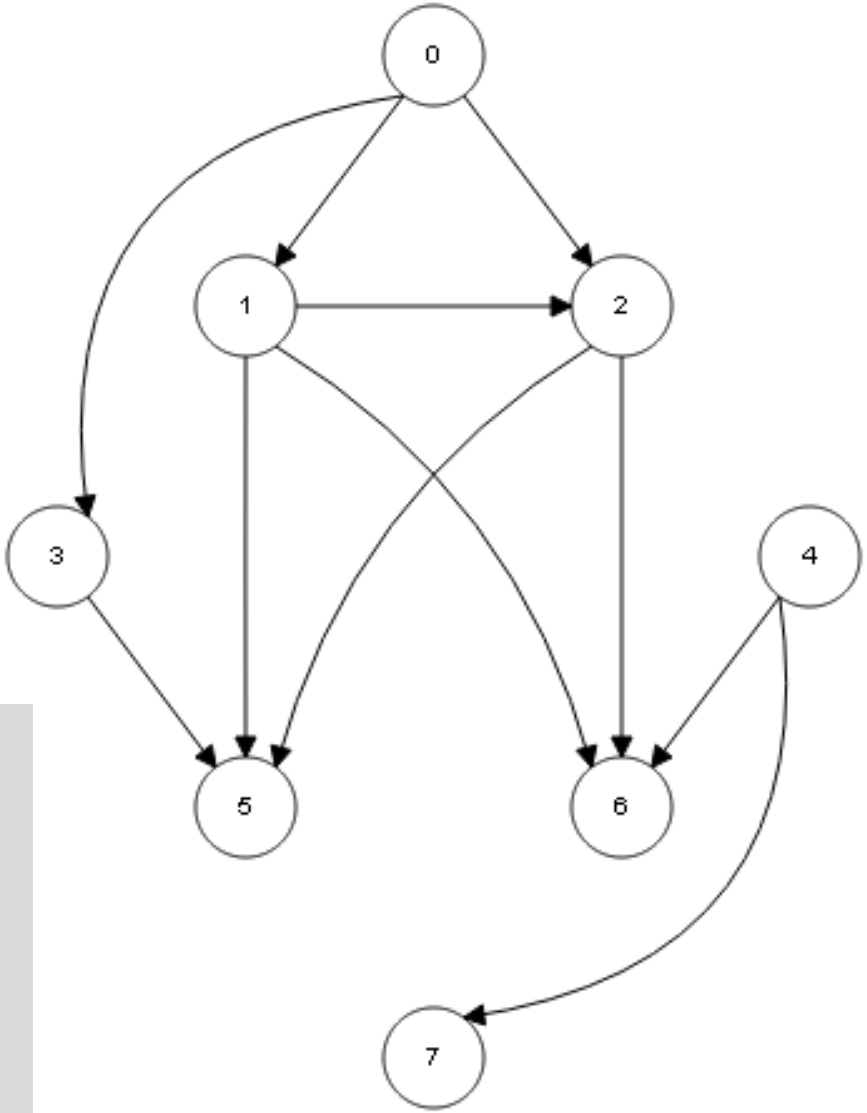

Algoritmo de Kahn

	I	S	L
0	0	0	4
1	1		
2	2		
3	1		
4	0		
5	3		
6	2		
7	0		

```
while S ≠ ∅
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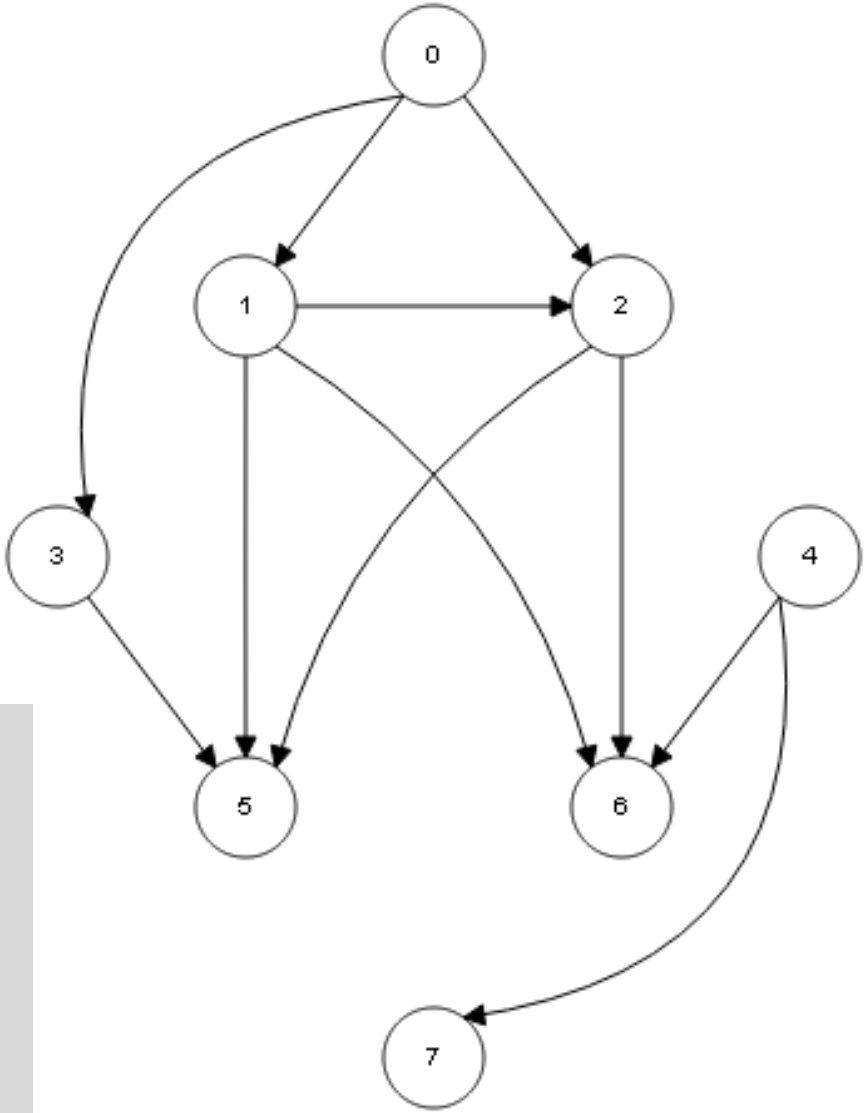
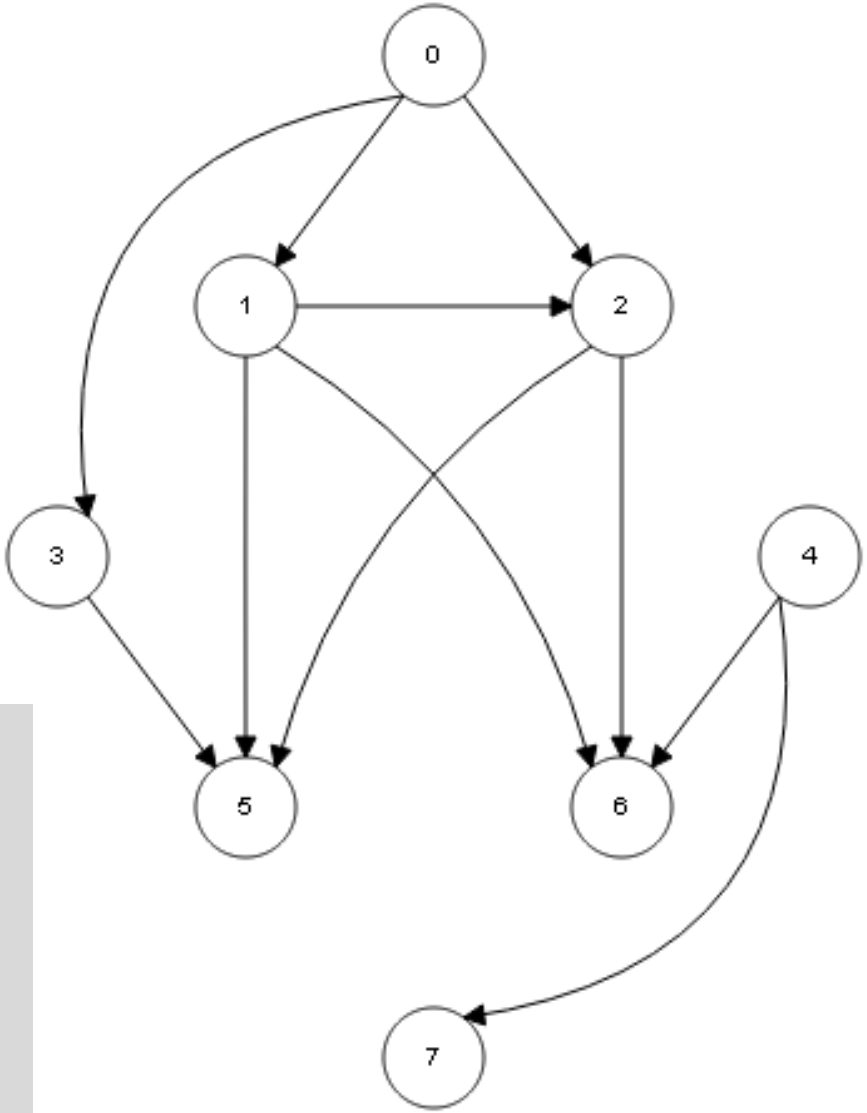


Algoritmo de Kahn

	I
0	0
1	1
2	2
3	1
4	0
5	3
6	2
7	0

S
0
7

L
4

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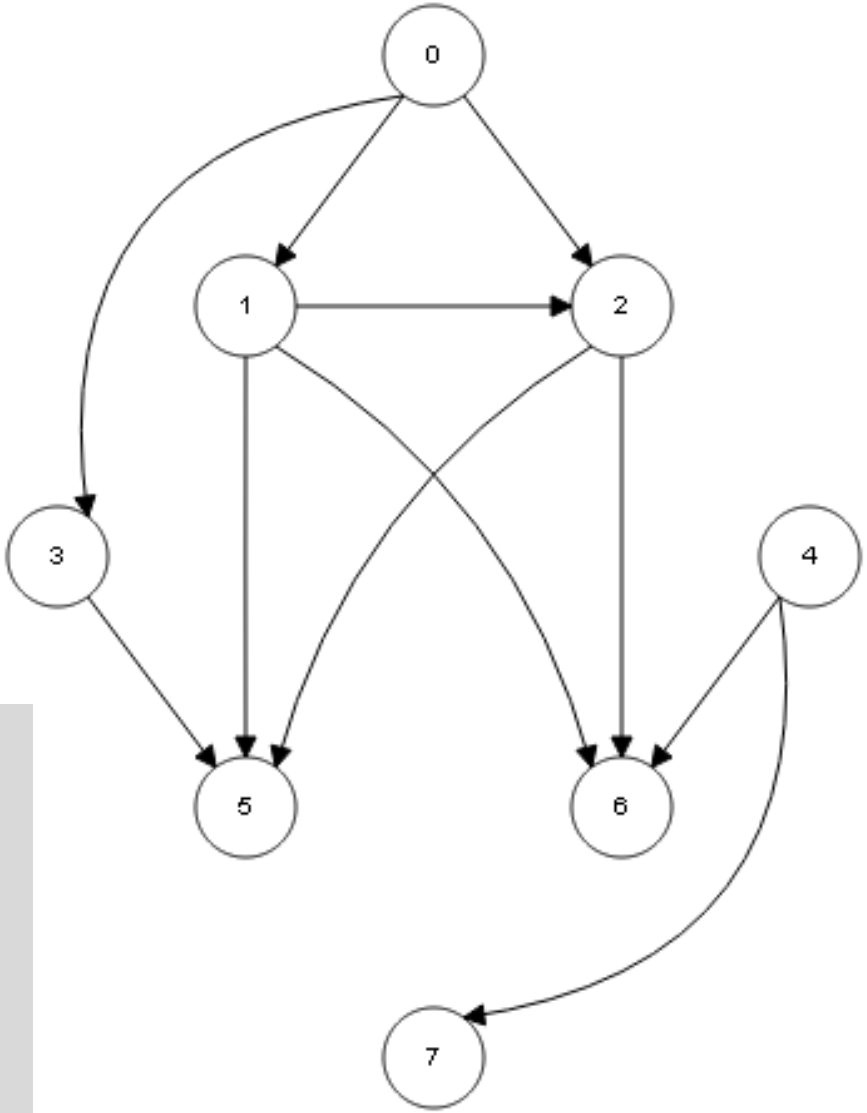
Algoritmo de Kahn

	I
0	0
1	1
2	2
3	1
4	0
5	3
6	2
7	0

S
0

L
4
7

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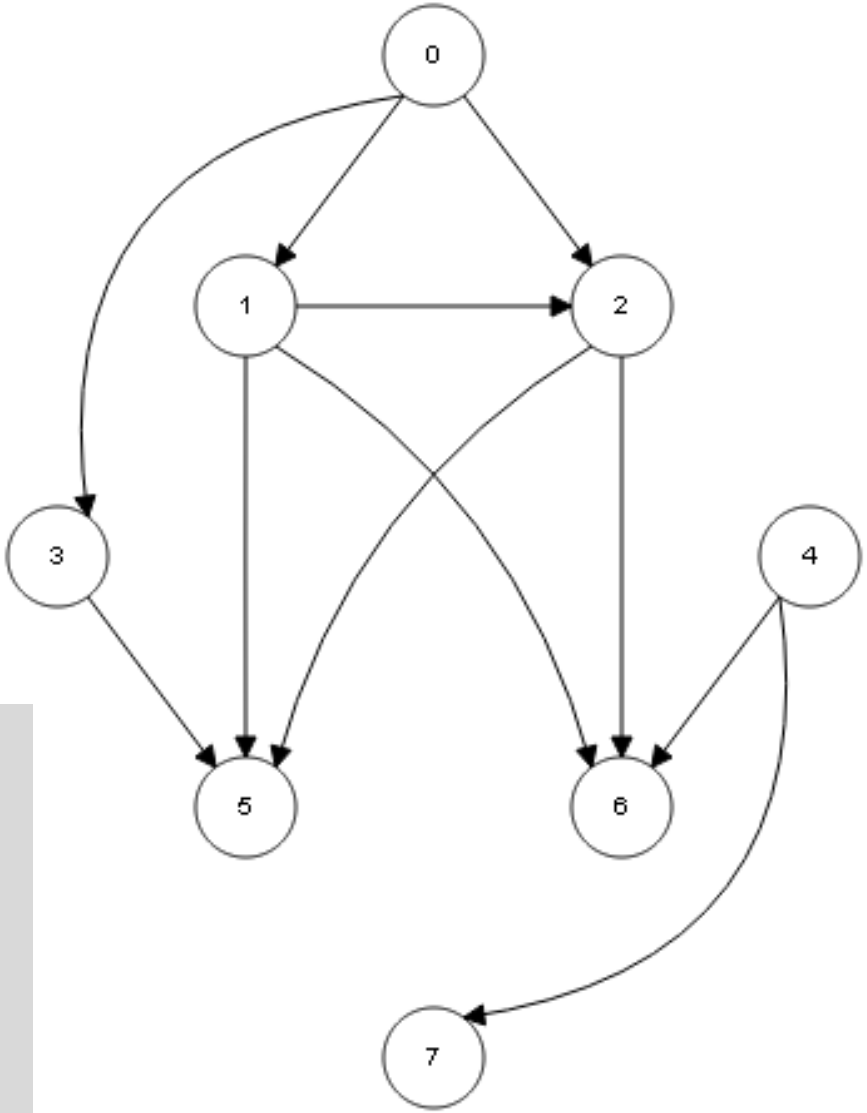
Algoritmo de Kahn

	I
0	0
1	1
2	2
3	1
4	0
5	3
6	2
7	0

S
/

L
4
7
0

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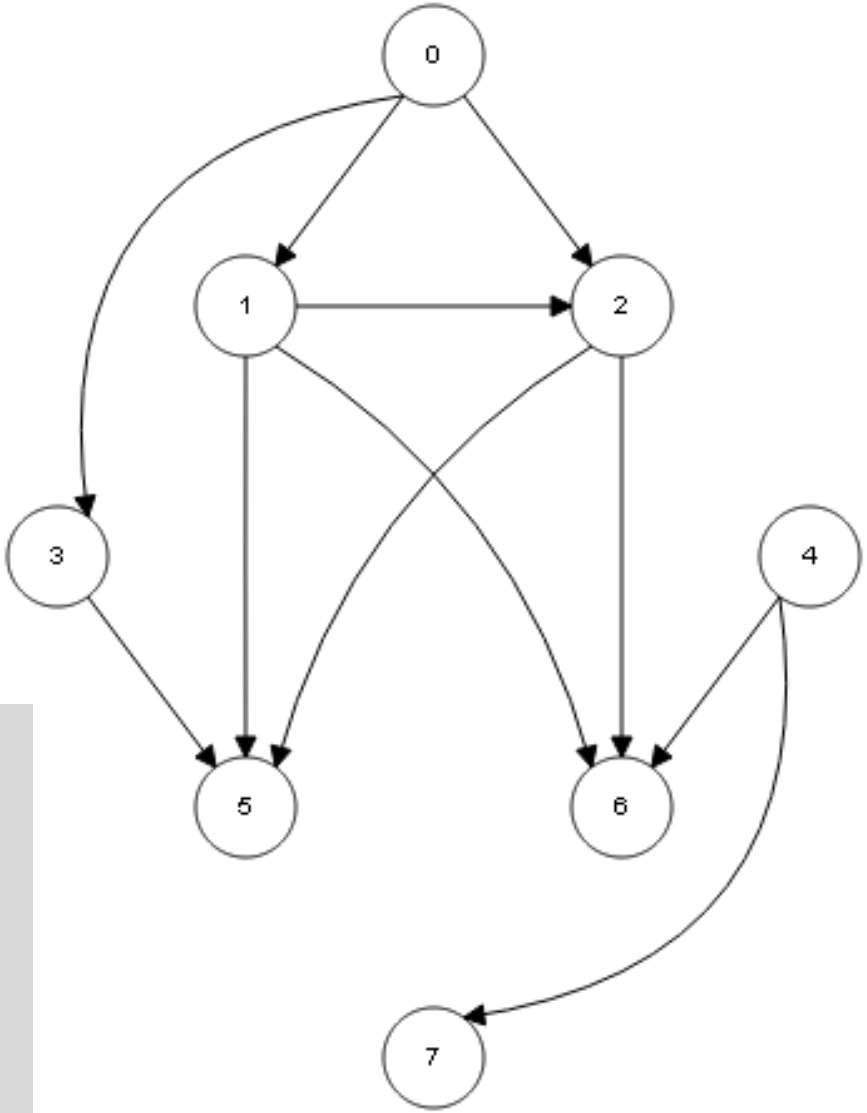
Algoritmo de Kahn

	I
0	0
1	1
2	2
3	1
4	0
5	3
6	2
7	0

S
/

L
4
7
0

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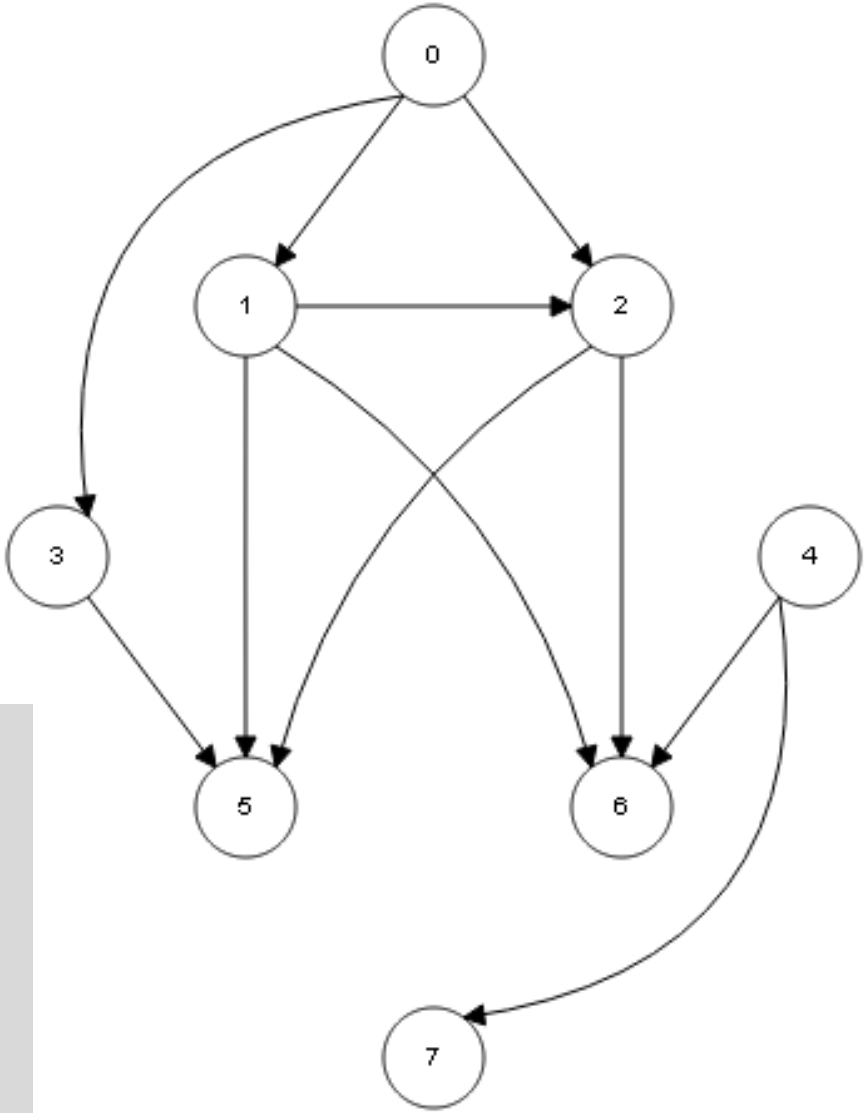
Algoritmo de Kahn

	I
0	0
1	0
2	1
3	0
4	0
5	3
6	2
7	0

S
/

L
4
7
0

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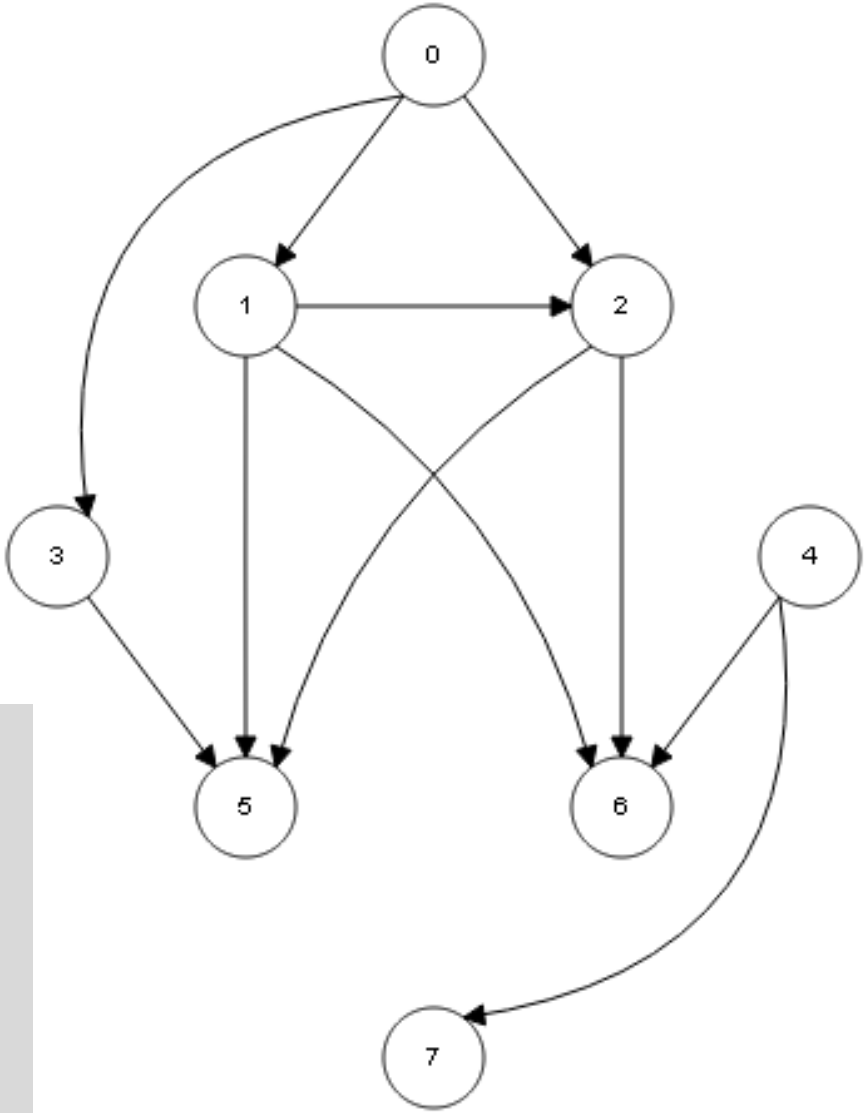
Algoritmo de Kahn

	I
0	0
1	0
2	1
3	0
4	0
5	3
6	2
7	0

S
1
3

L
4
7
0

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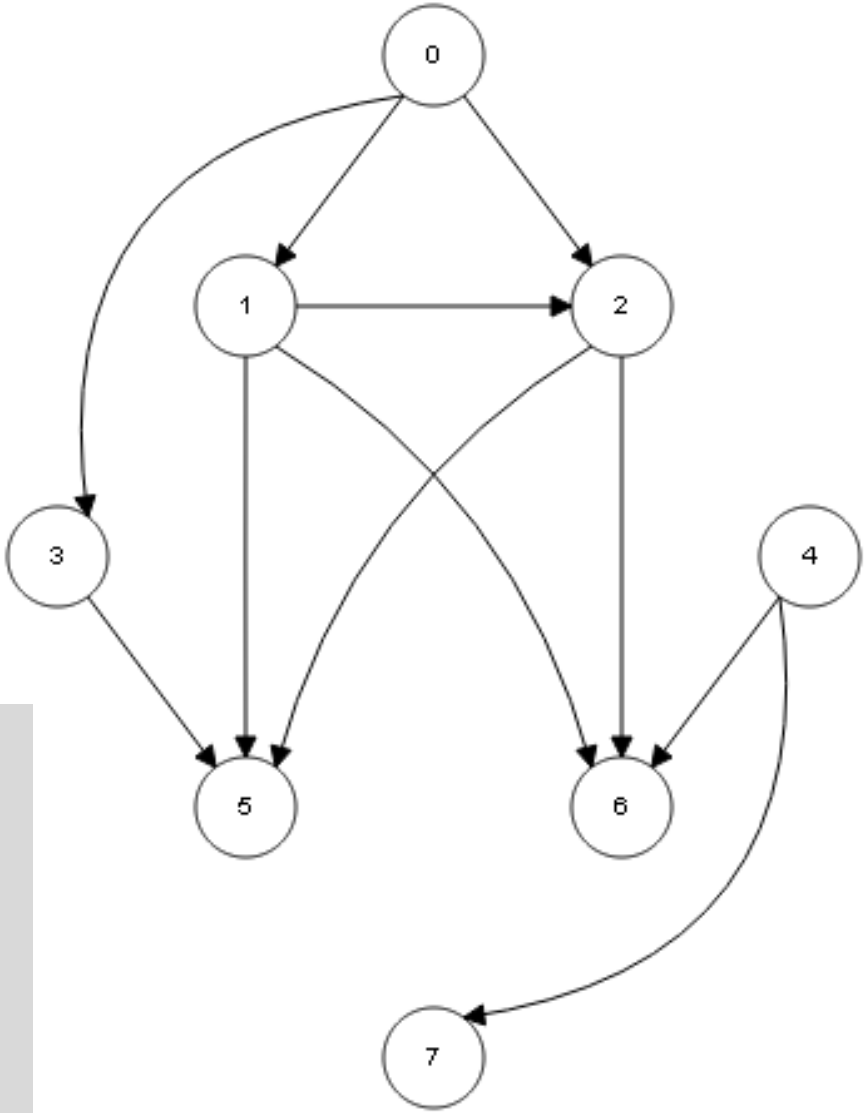
Algoritmo de Kahn

	I
0	0
1	0
2	1
3	0
4	0
5	3
6	2
7	0

S
1

L
4
7
0
3

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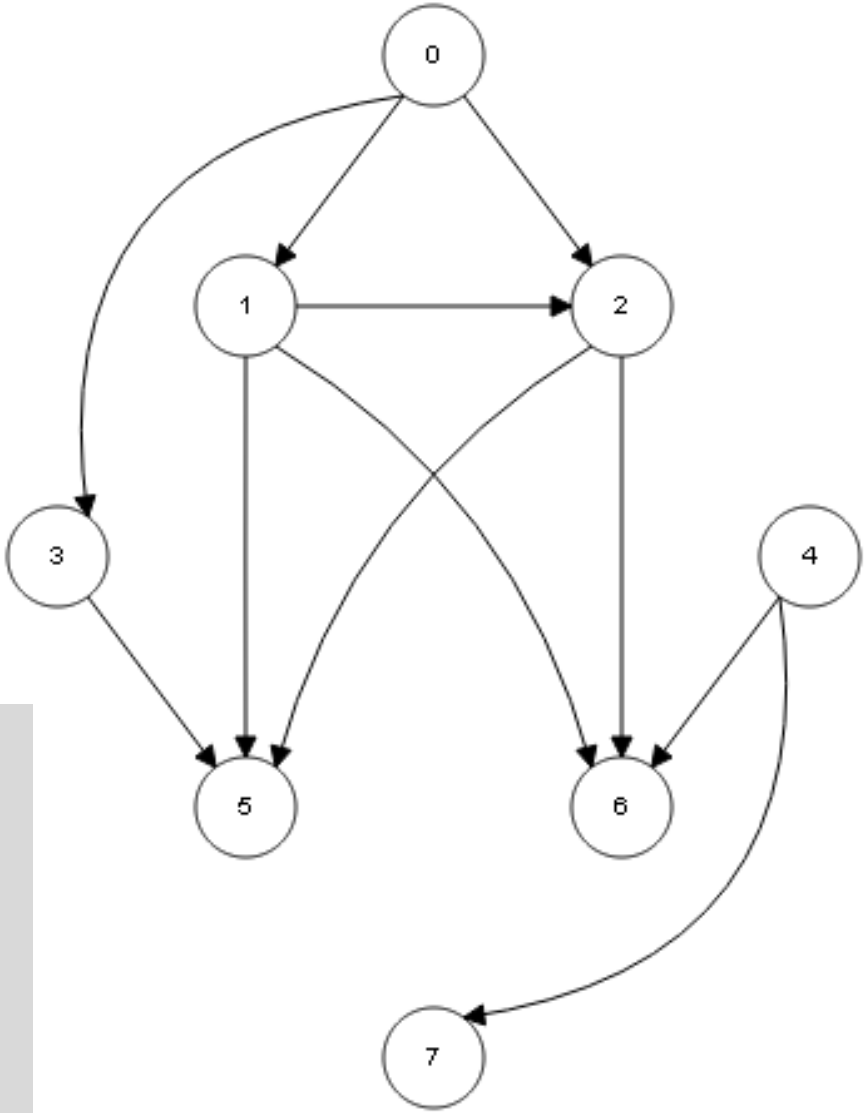
Algoritmo de Kahn

	I
0	0
1	0
2	1
3	0
4	0
5	3
6	2
7	0

S
1

L
4
7
0
3

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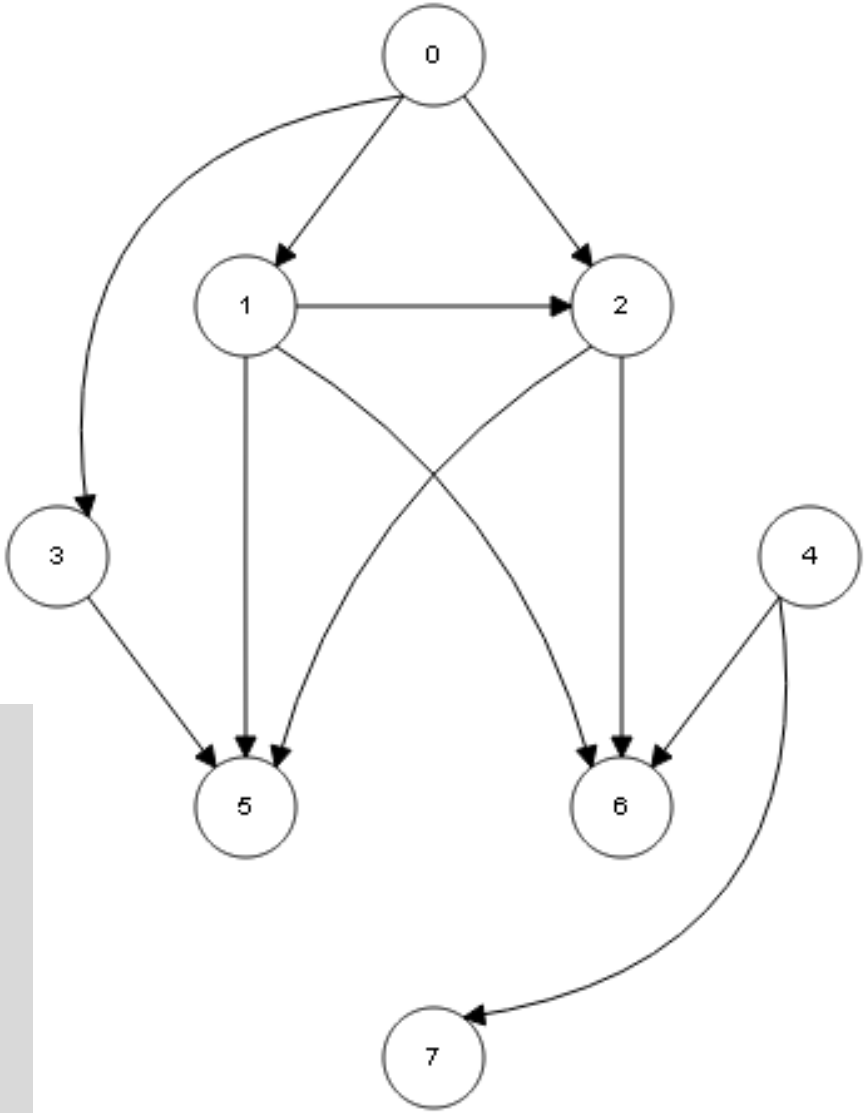
Algoritmo de Kahn

	I
0	0
1	0
2	1
3	0
4	0
5	2
6	2
7	0

S
1

L
4
7
0
3

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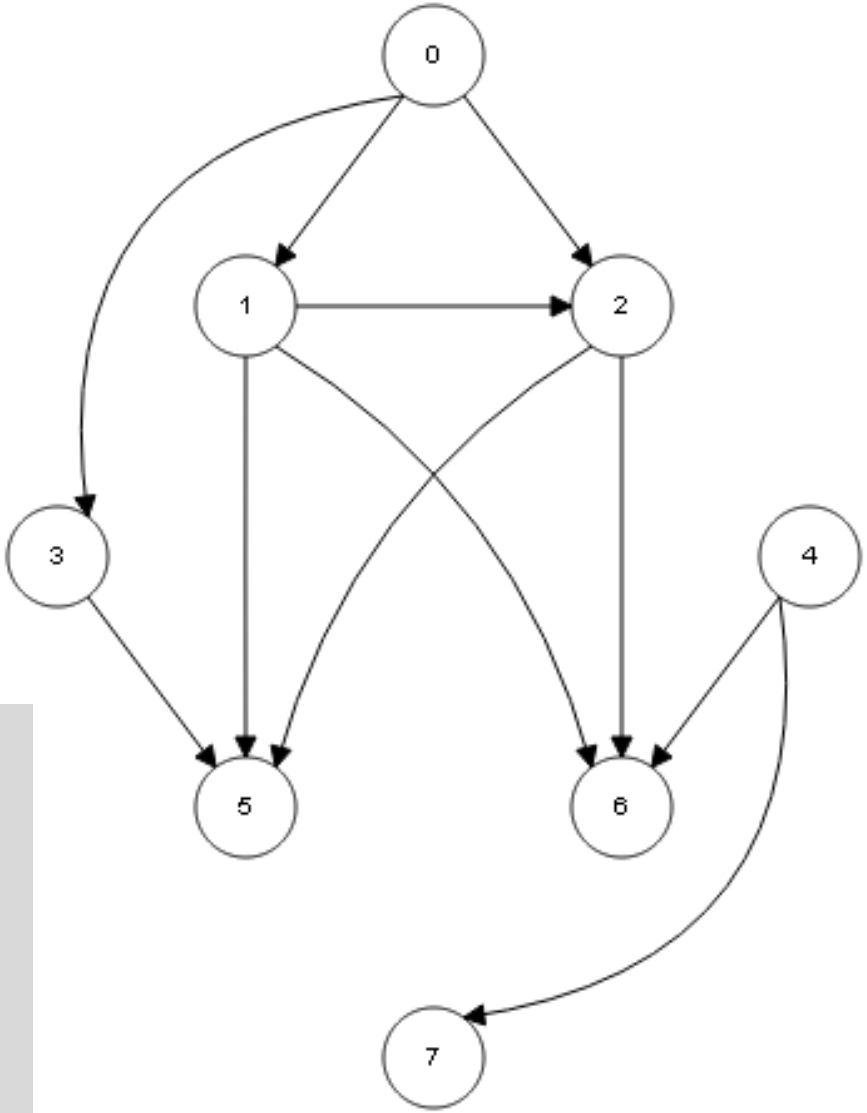
Algoritmo de Kahn

	I
0	0
1	0
2	1
3	0
4	0
5	2
6	2
7	0

S
/

L
4
7
0
3
1

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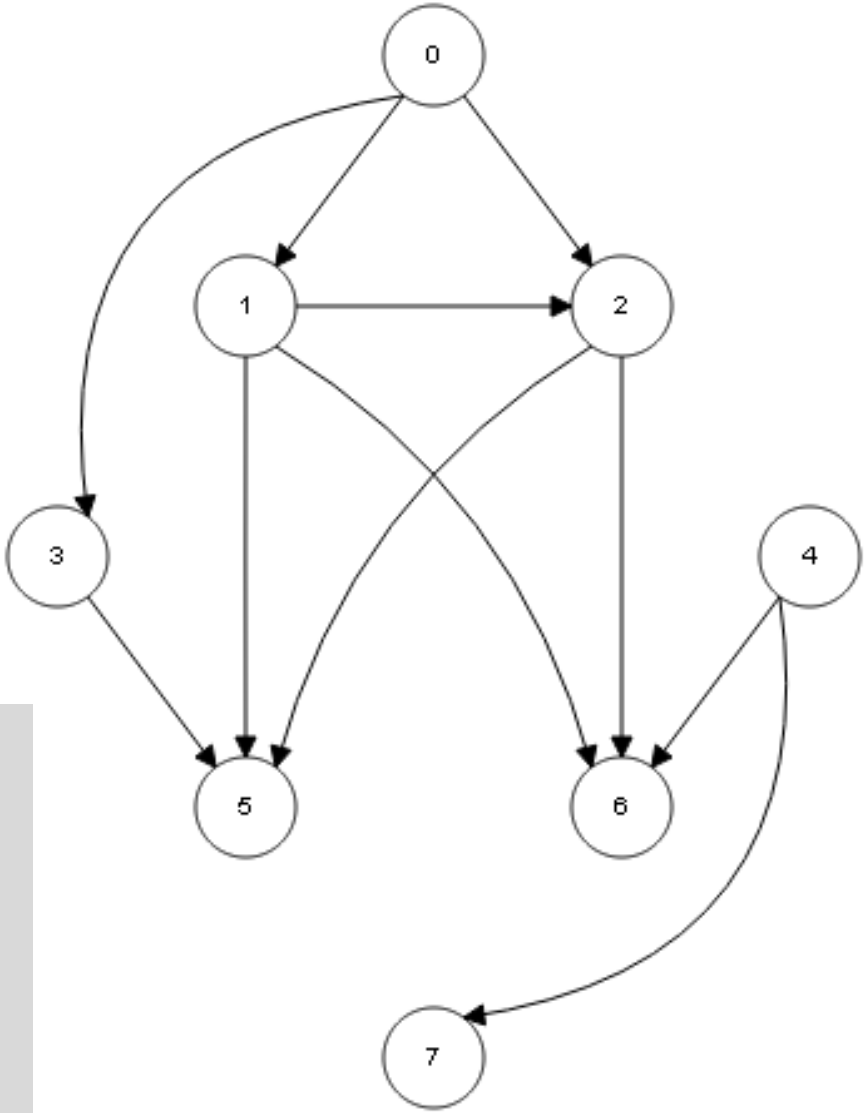
Algoritmo de Kahn

	I
0	0
1	0
2	1
3	0
4	0
5	2
6	2
7	0

S
/

L
4
7
0
3
1

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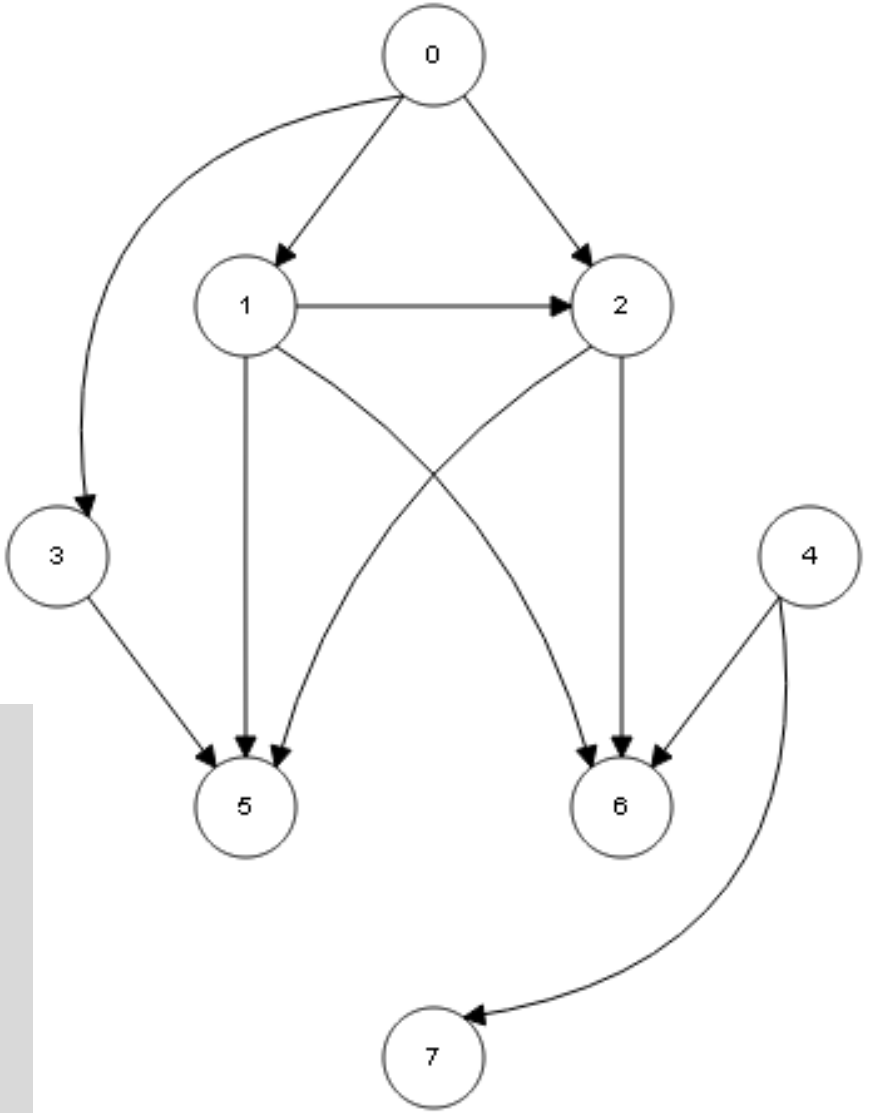
Algoritmo de Kahn

	I
0	0
1	0
2	0
3	0
4	0
5	1
6	1
7	0

S
/

L
4
7
0
3
1

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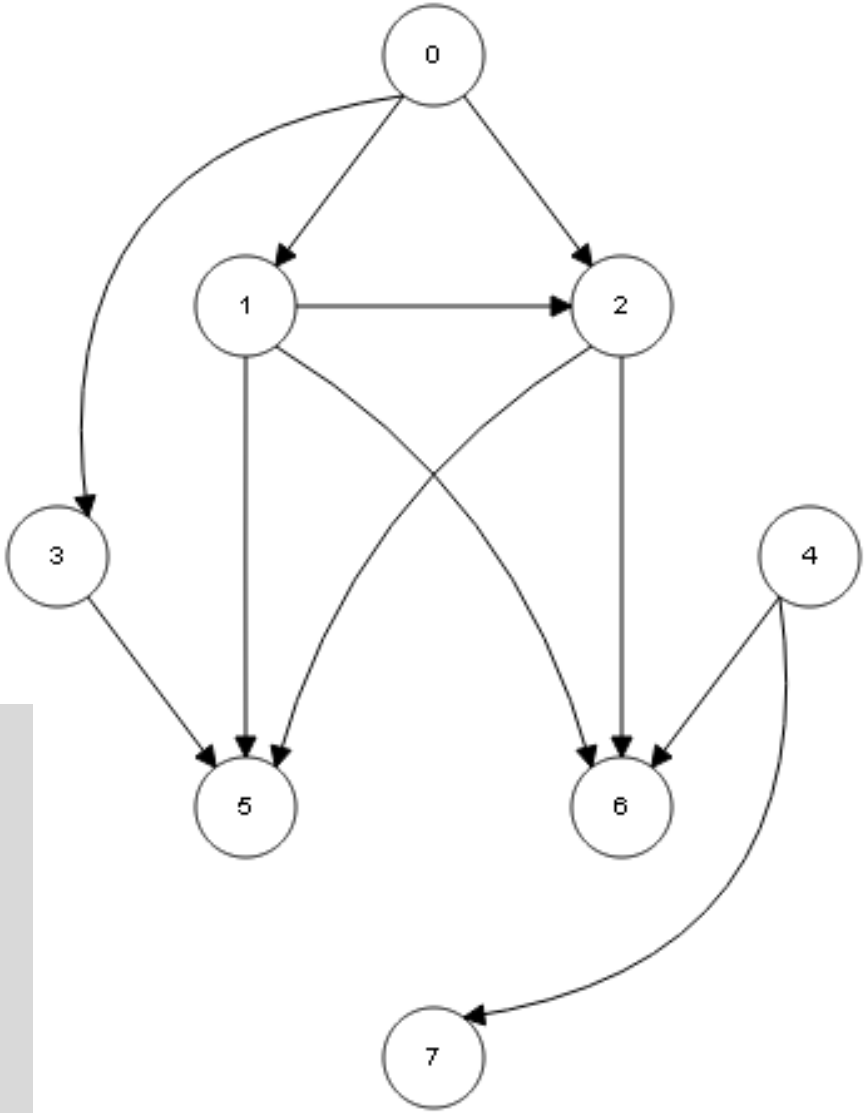
```

Algoritmo de Kahn

	I
0	0
1	0
2	0
3	0
4	0
5	1
6	1
7	0

S
2

L
4
7
0
3
1

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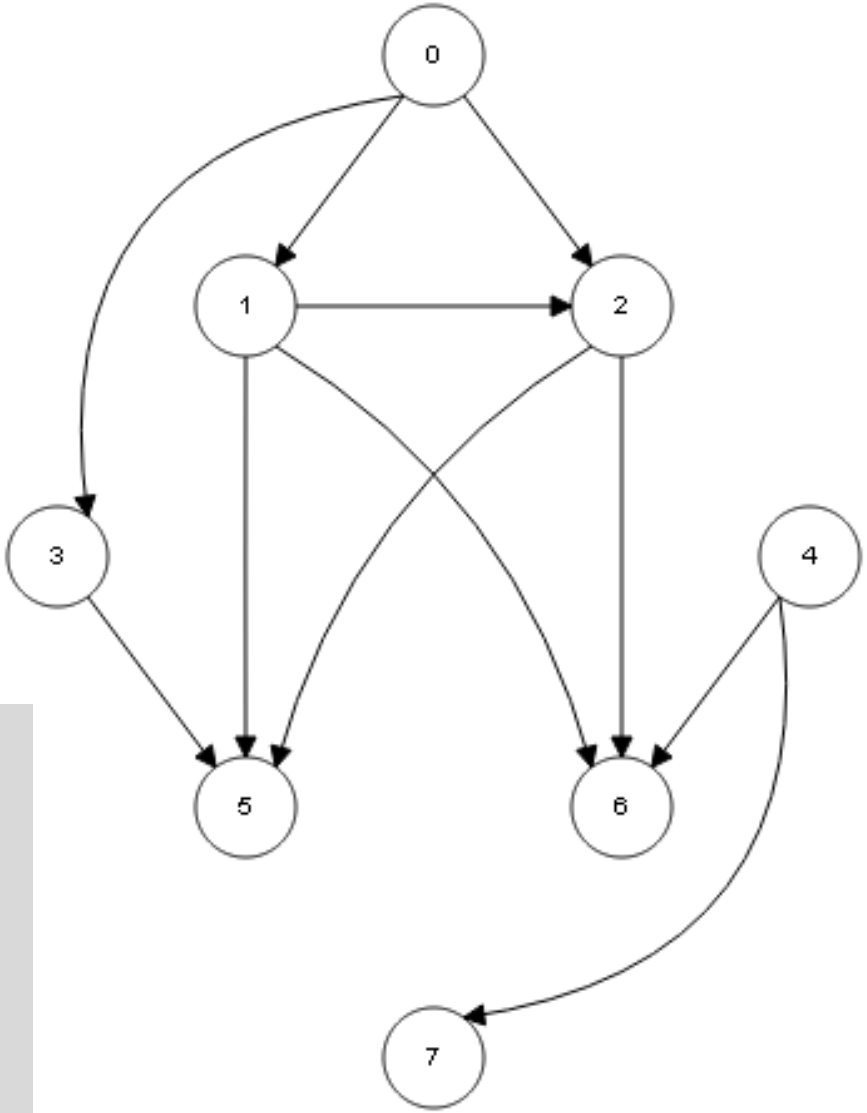
Algoritmo de Kahn

	I
0	0
1	0
2	0
3	0
4	0
5	1
6	1
7	0

S
/

L
4
7
0
3
1
2

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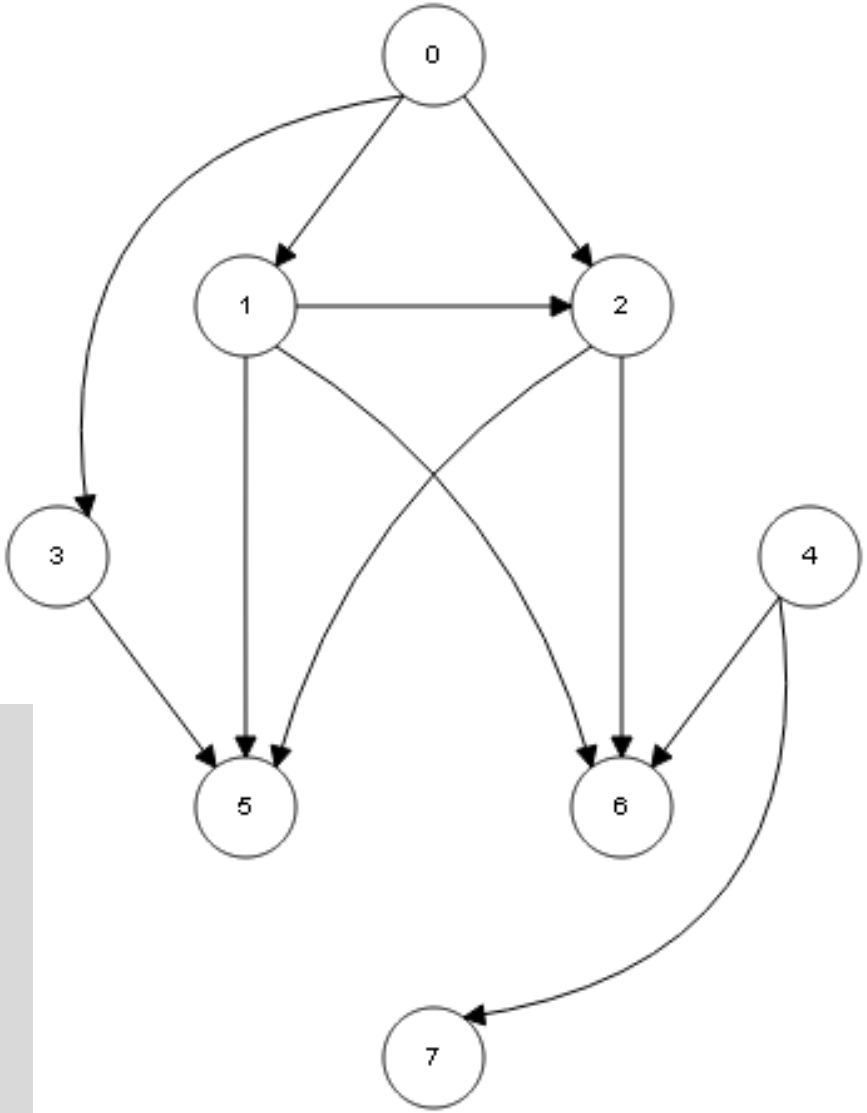
```

Algoritmo de Kahn

	I
0	0
1	0
2	0
3	0
4	0
5	1
6	1
7	0

S
/

L
4
7
0
3
1
2

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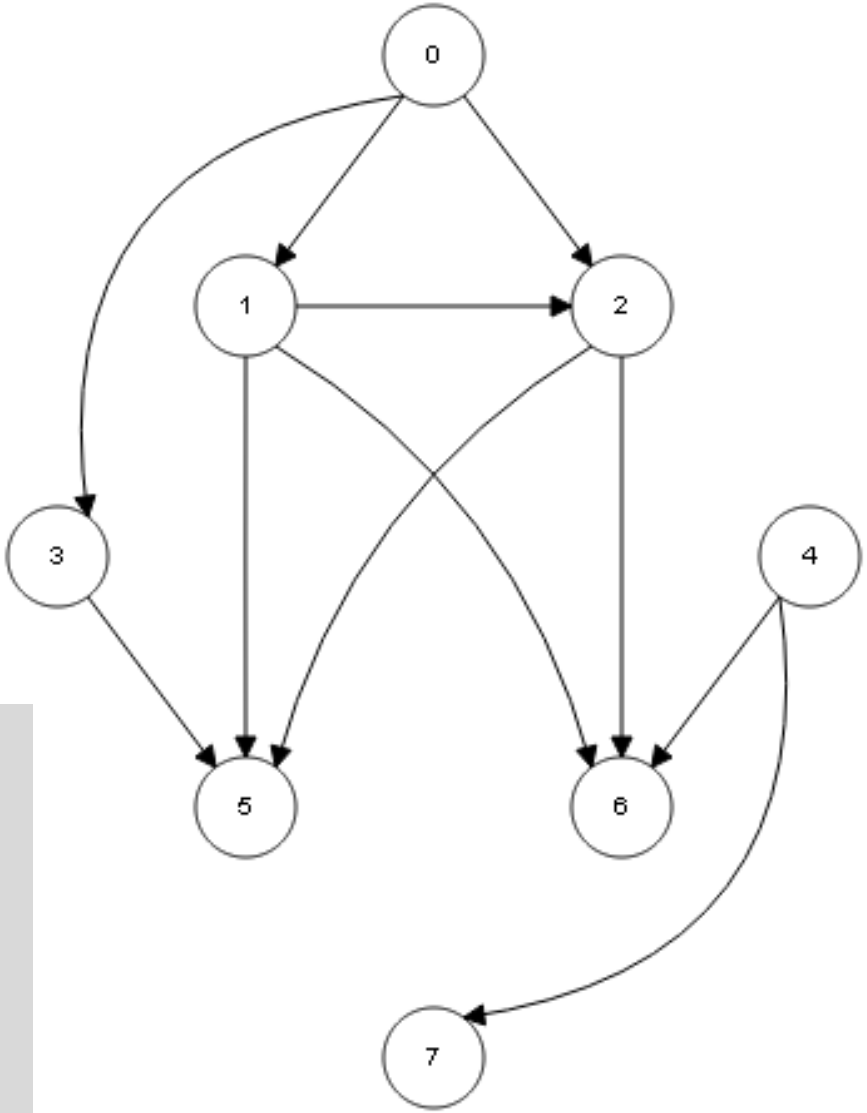
Algoritmo de Kahn

	I
0	0
1	0
2	0
3	0
4	0
5	0
6	0
7	0

S
/

L
4
7
0
3
1
2

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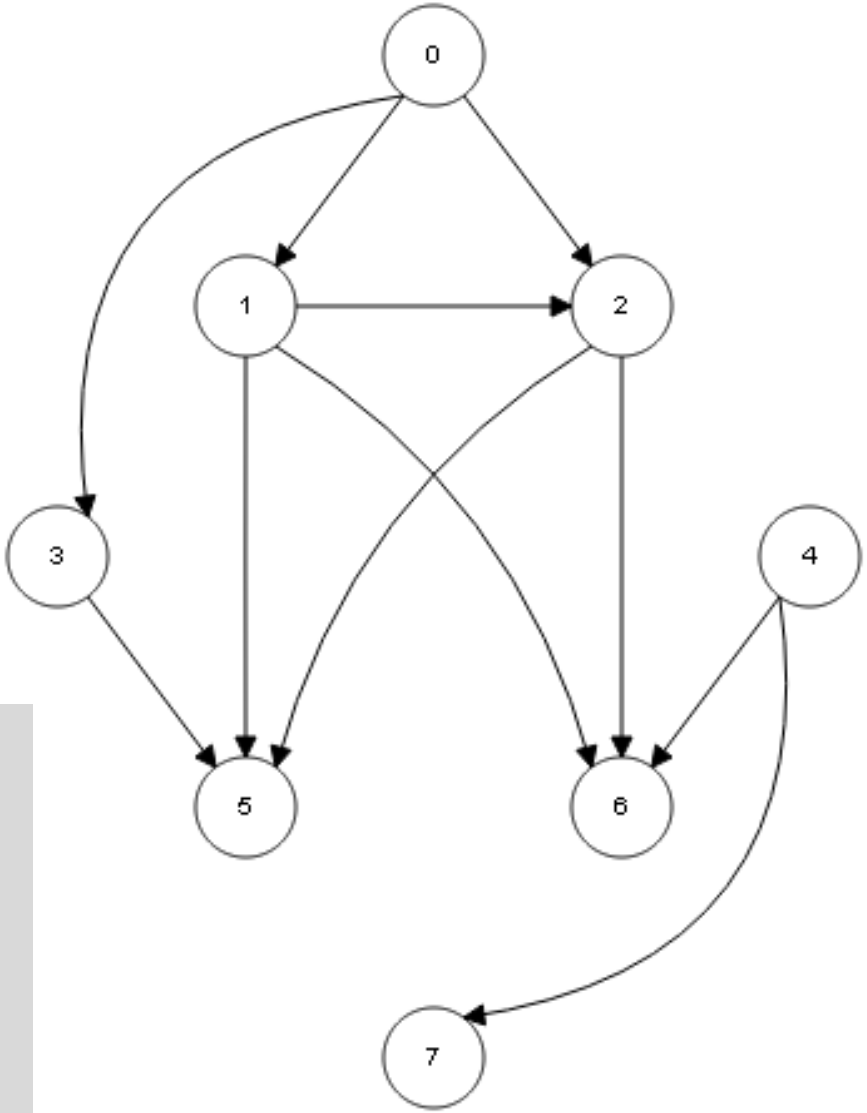
Algoritmo de Kahn

	I
0	0
1	0
2	0
3	0
4	0
5	0
6	0
7	0

S
5
6

L
4
7
0
3
1
2

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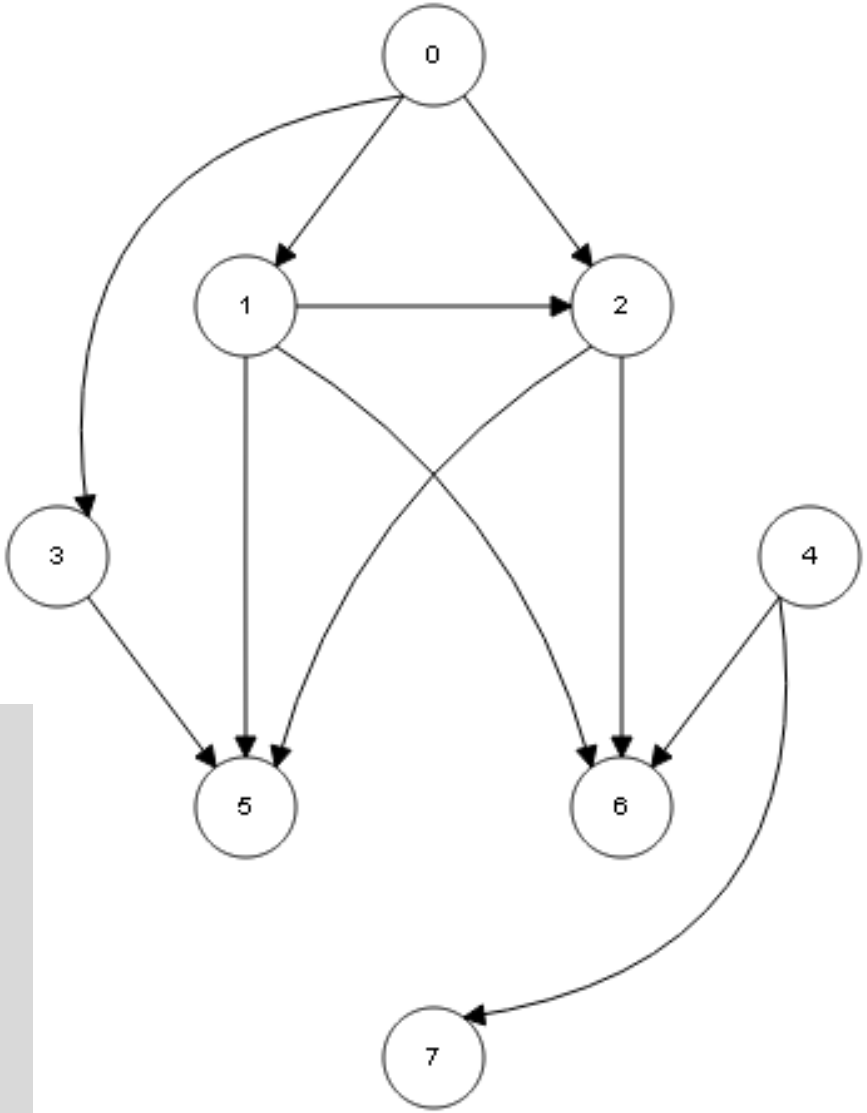
```

Algoritmo de Kahn

	I
0	0
1	0
2	0
3	0
4	0
5	0
6	0
7	0

S
5

L
4
7
0
3
1
2
6

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Algoritmo de Kahn

	I
0	0
1	0
2	0
3	0
4	0
5	0
6	0
7	0

S
/

L
4
7
0
3
1
2
6
5

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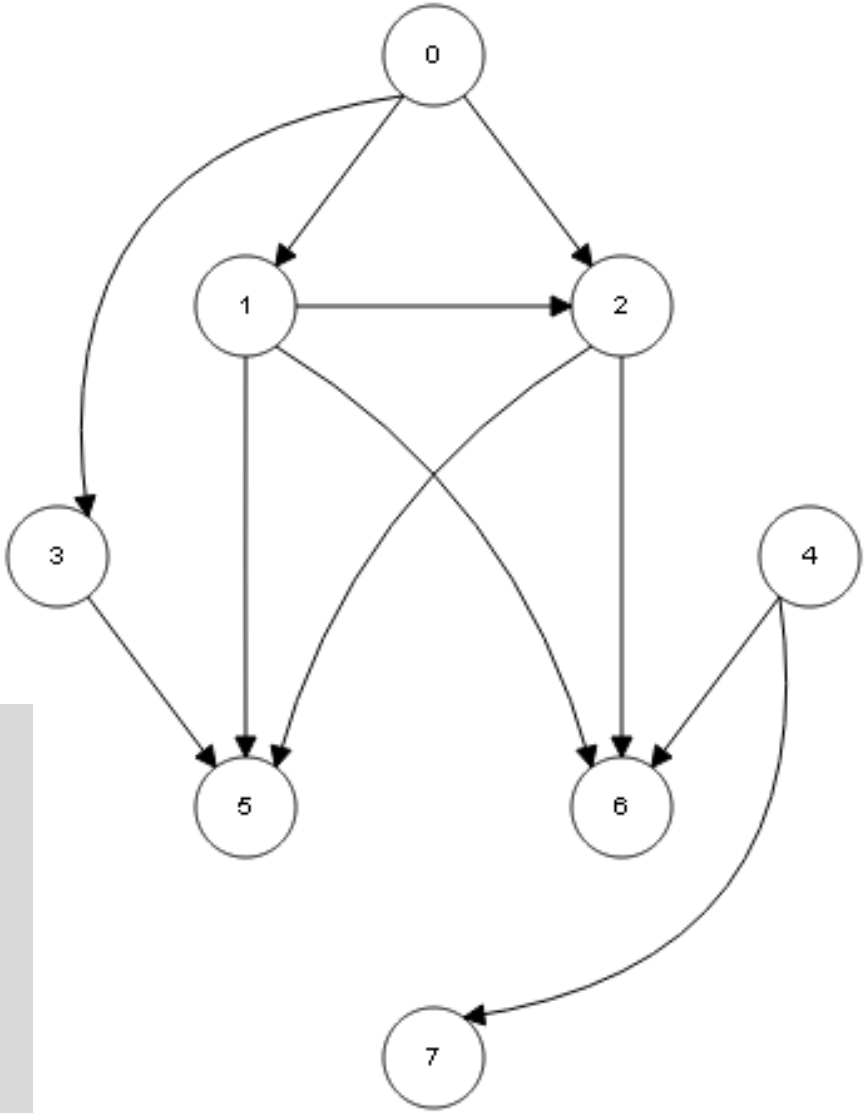
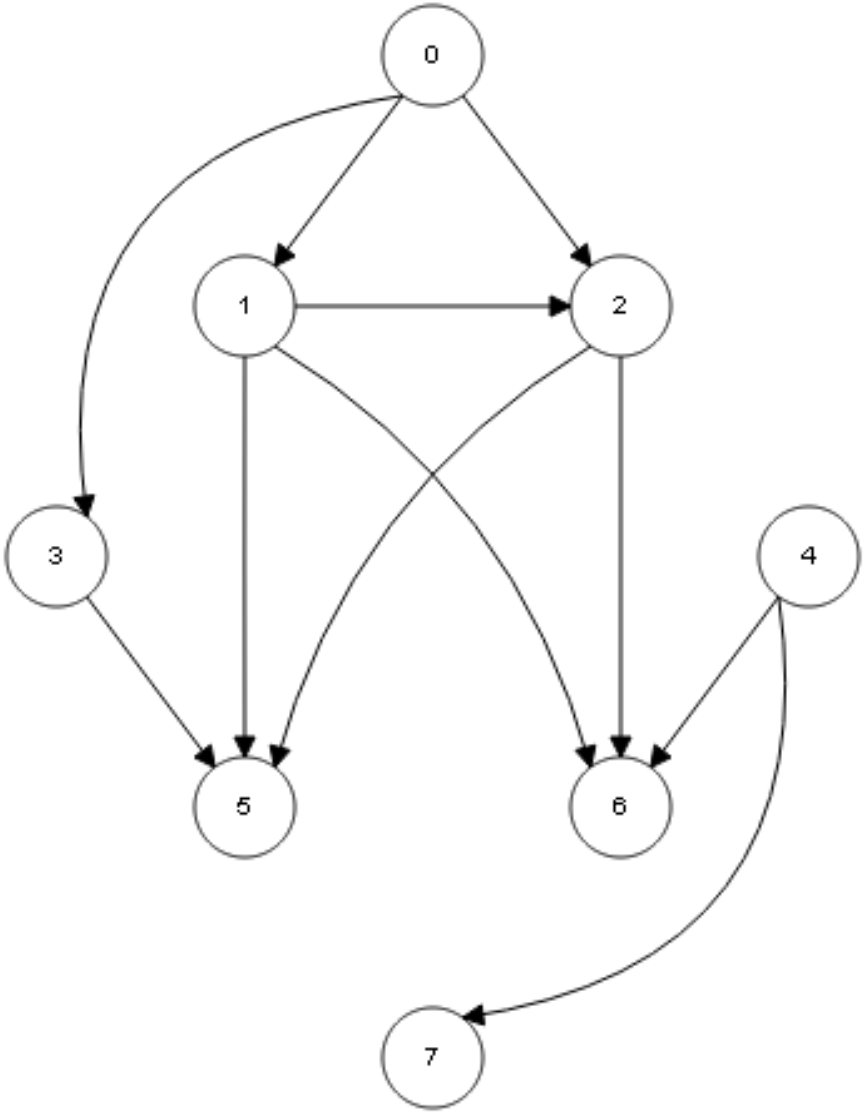


Algoritmo de Kahn

	I
0	0
1	0
2	0
3	0
4	0
5	0
6	0
7	0

S
/

L
4
7
0
3
1
2
6
5



Algoritmo de Kahn – Análise

```
OrdenacaoTopologicaKahn(G)
  L ← ∅
  for each uv ∈ A[G]
    I[v] ← I[v] + 1
  S ← vertices com grau de
      entrada zero (S = pilha)
  while S ≠ ∅
    v ← unstack(S)
    stack(v, L)
    for each u ∈ Adj[v]
      I[u] ← I[u] - 1
      if I[u] = 0
        stack(u, S)
  return L
```

- Inicializar o vetor `I`: O(A)
- Inicializar a pilha S com os vértices de entrada zero: O(A)
- Desempilhar e Empilhar vértices: O(V) vertices, cada um com tempo O(1)
- Reduzir `I` para todos os vértices adjacentes: O(V)
- Complexidade: **O(V + A)**

Algoritmo com Busca em Profundidade

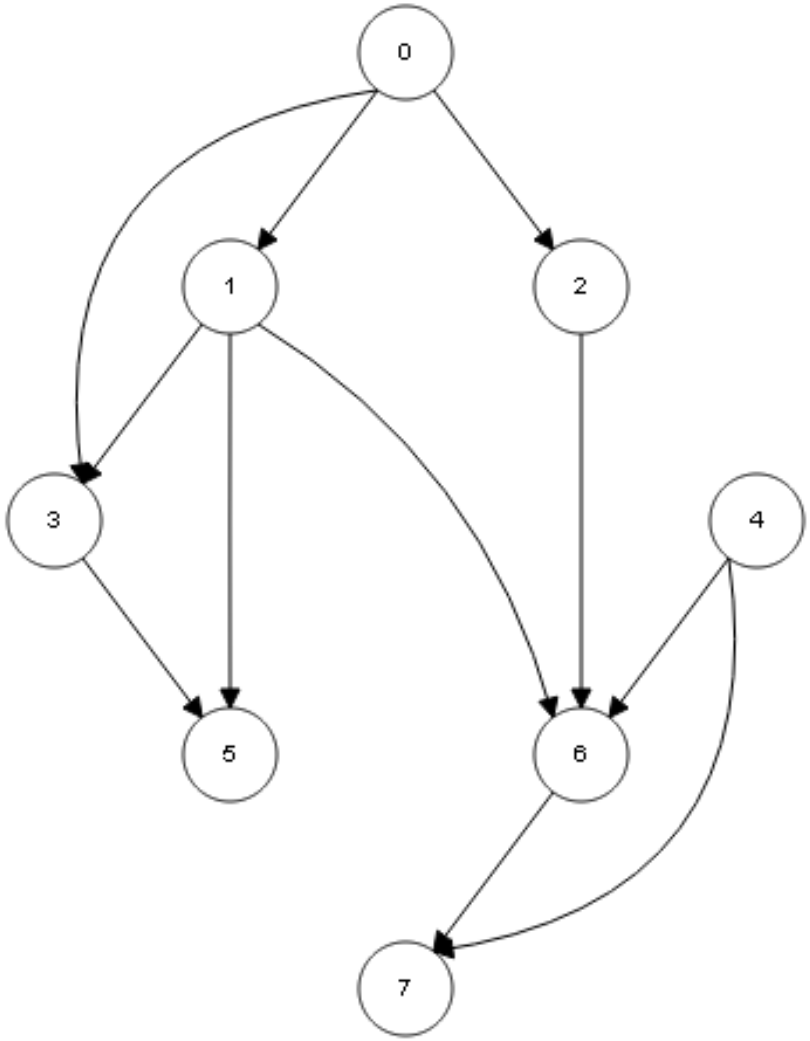
```
OrdenacaoTopologicaDFS(G)
  L ← ∅
  BuscaEmProfundidade(G)
  return L
```

```
BuscaEmProfundidade(G)
  for each u ∈ V[G]
    c[u] ← white
    π[u] ← NULL
  time ← 0
  for each u ∈ V[G]
    if c[u] = white
      visita(u)
```

```
visita(u)
  c[u] ← gray
  d[u] ← time ← time + 1
  for each v ∈ Adj[u]
    if c[v] = white
      π[v] ← u
      visita(v)
  c[u] ← black
  f[u] ← time ← time + 1
  insertFront(L, u)
```

Ordenação Topológica – DFS

L | /



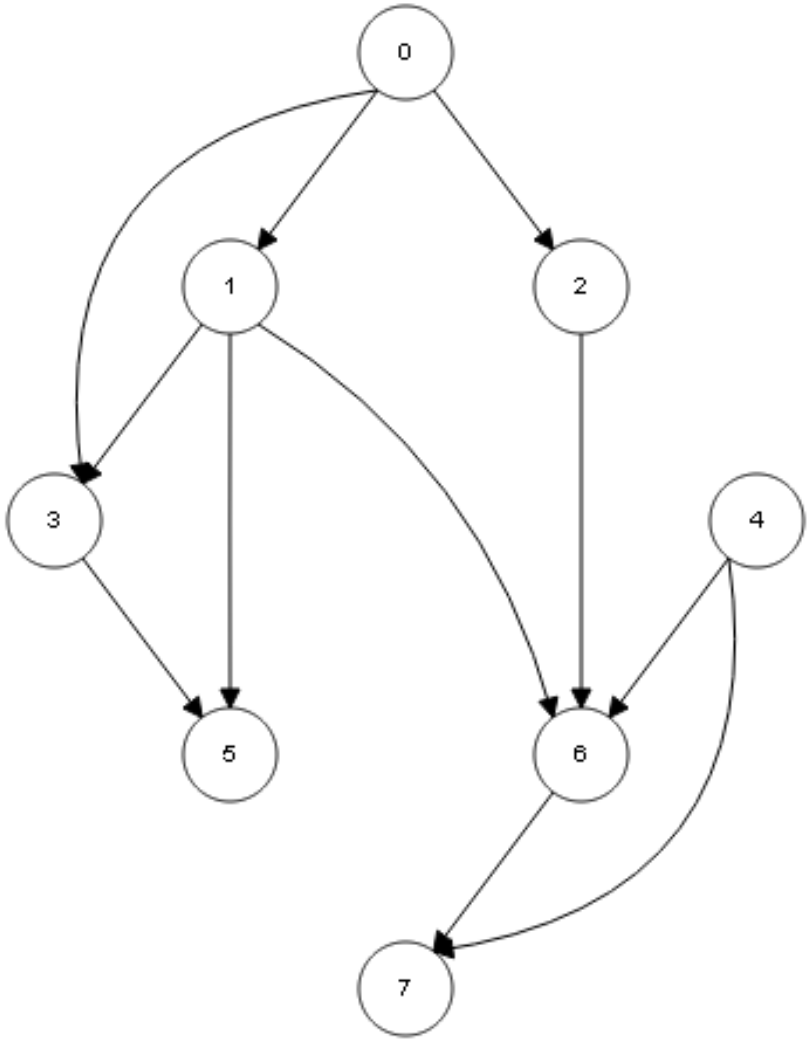

```
OrdenacaoTopologicaDFS(G)
  L ← ∅
  BuscaEmProfundidade(G)
  return L
```

Ordenação Topológica – DFS

L | /

	0	1	2	3	4	5	6	7
c	w	w	w	w	w	w	w	w
d								
f								

```
c[u] ← gray
d[u] ← time ← time + 1
for each v ∈ Adj[u]
  if c[v] = white
    π[v] ← u
    visita(v)
c[u] ← black
f[u] ← time ← time + 1
insertFront(L, u)
```

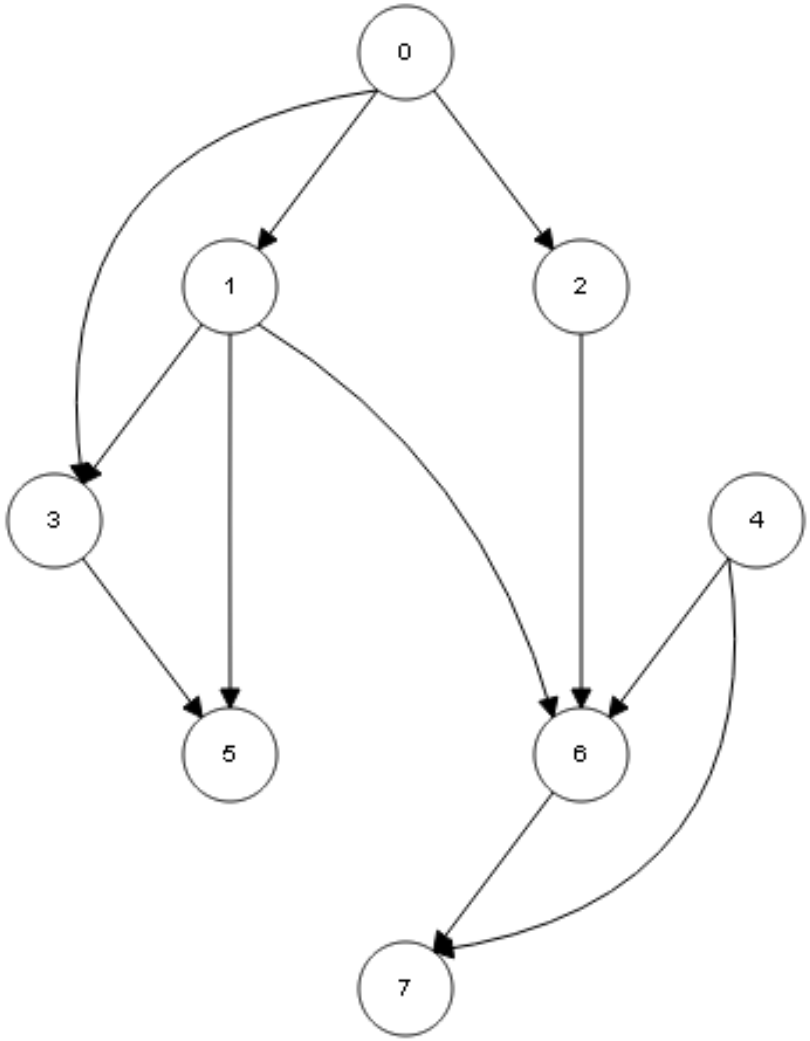


Ordenação Topológica – DFS

L | /

	0	1	2	3	4	5	6	7
c	g	w	w	w	w	w	w	w
d	1							
f								

```
c[u] ← gray
d[u] ← time ← time + 1
for each v ∈ Adj[u]
  if c[v] = white
    π[v] ← u
    visita(v)
c[u] ← black
f[u] ← time ← time + 1
insertFront(L, u)
```

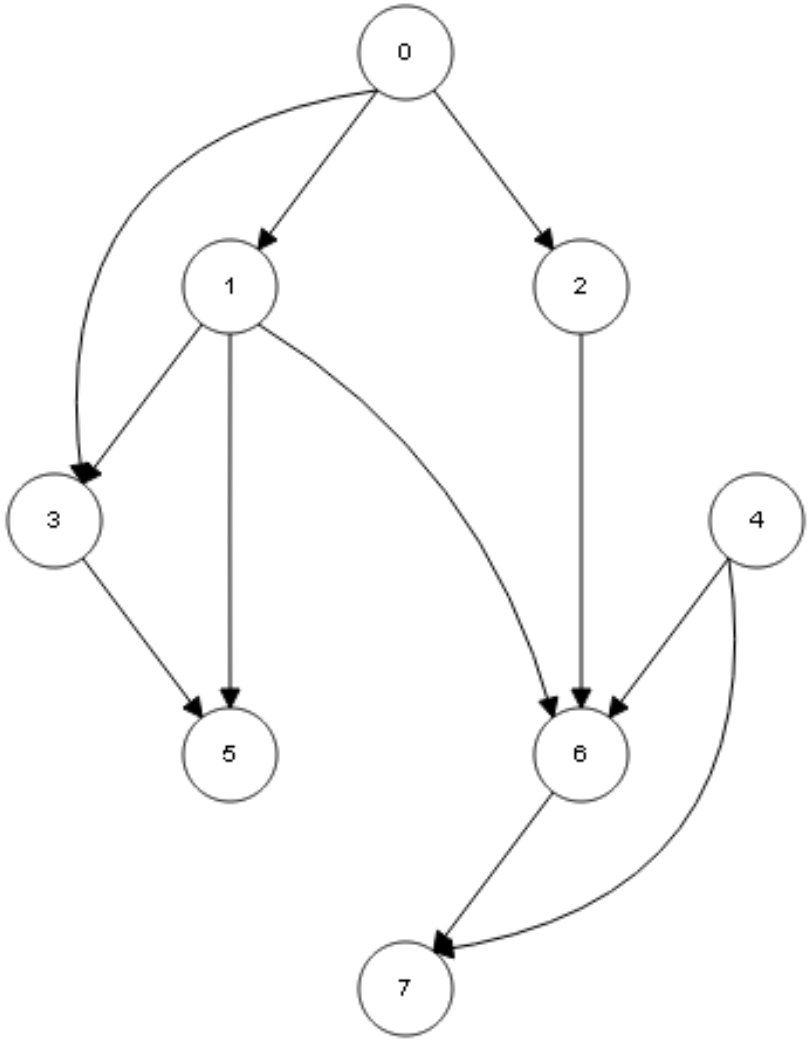


Ordenação Topológica – DFS

L | /

	0	1	2	3	4	5	6	7
c	g	g	w	w	w	w	w	w
d	1	2						
f								

```
c[u] ← gray
d[u] ← time ← time + 1
for each v ∈ Adj[u]
  if c[v] = white
    π[v] ← u
    visita(v)
c[u] ← black
f[u] ← time ← time + 1
insertFront(L, u)
```

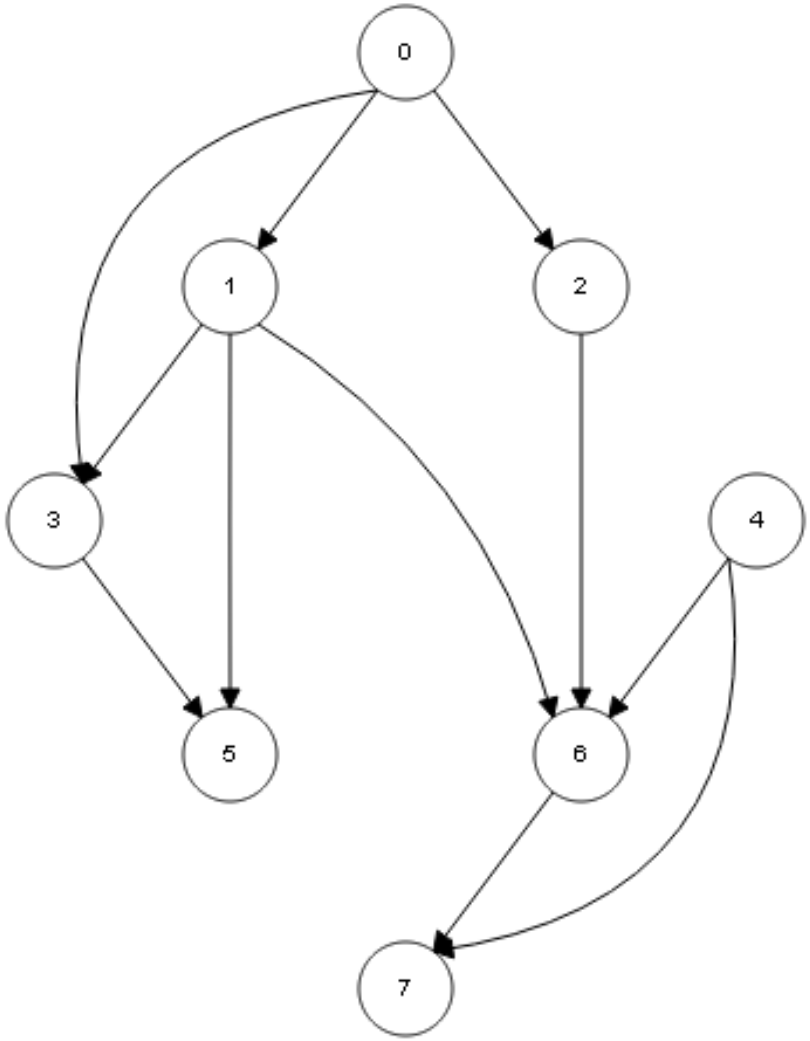


Ordenação Topológica – DFS

L | / |

	0	1	2	3	4	5	6	7
c	g	g	w	g	w	w	w	w
d	1	2		3				
f								

```
c[u] ← gray
d[u] ← time ← time + 1
for each v ∈ Adj[u]
  if c[v] = white
    π[v] ← u
    visita(v)
c[u] ← black
f[u] ← time ← time + 1
insertFront(L, u)
```

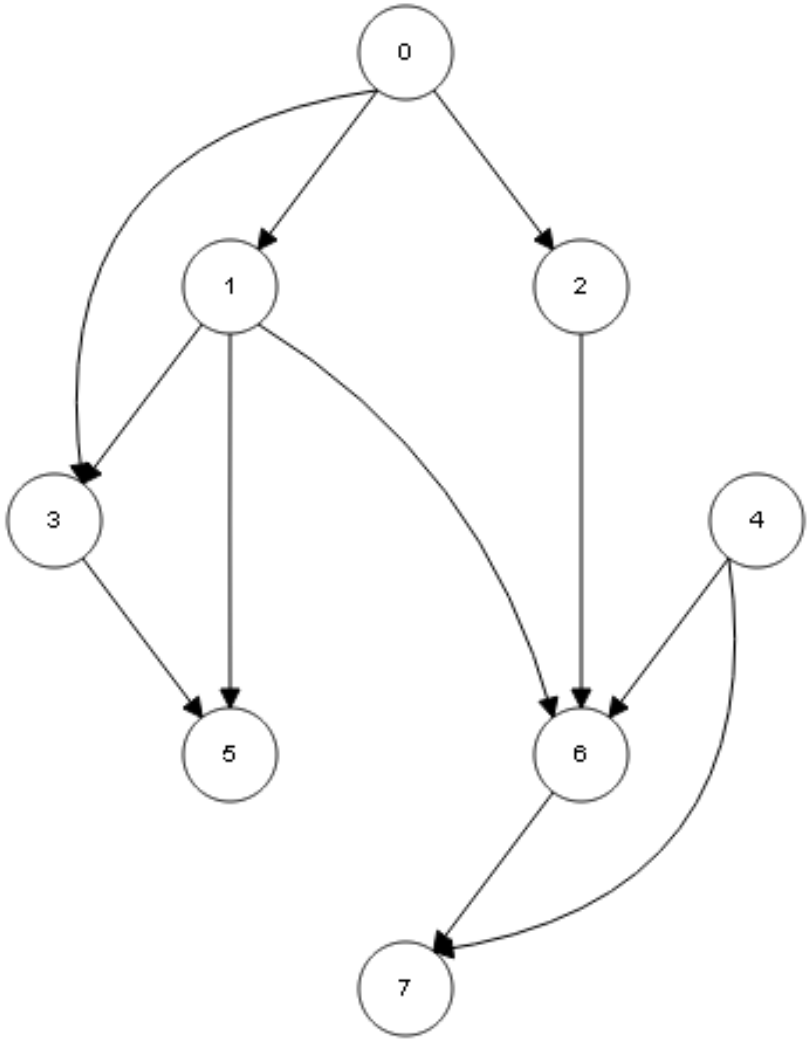


Ordenação Topológica – DFS

L | / |

	0	1	2	3	4	5	6	7
c	g	g	w	g	w	g	w	w
d	1	2		3		4		
f								

```
c[u] ← gray
d[u] ← time ← time + 1
for each v ∈ Adj[u]
  if c[v] = white
    π[v] ← u
    visita(v)
c[u] ← black
f[u] ← time ← time + 1
insertFront(L, u)
```

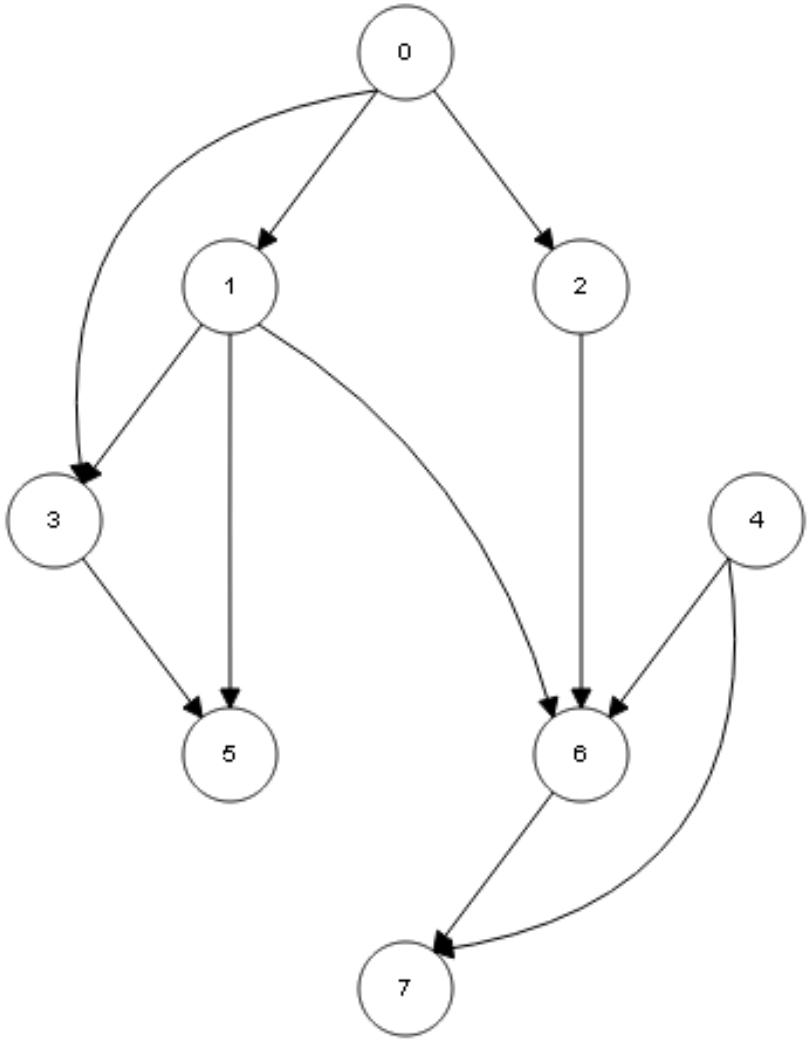


Ordenação Topológica – DFS

L | /

	0	1	2	3	4	5	6	7
c	g	g	w	g	w	b	w	w
d	1	2		3		4		
f						5		

```
c[u] ← gray
d[u] ← time ← time + 1
for each v ∈ Adj[u]
  if c[v] = white
    π[v] ← u
    visita(v)
c[u] ← black
f[u] ← time ← time + 1
insertFront(L, u)
```

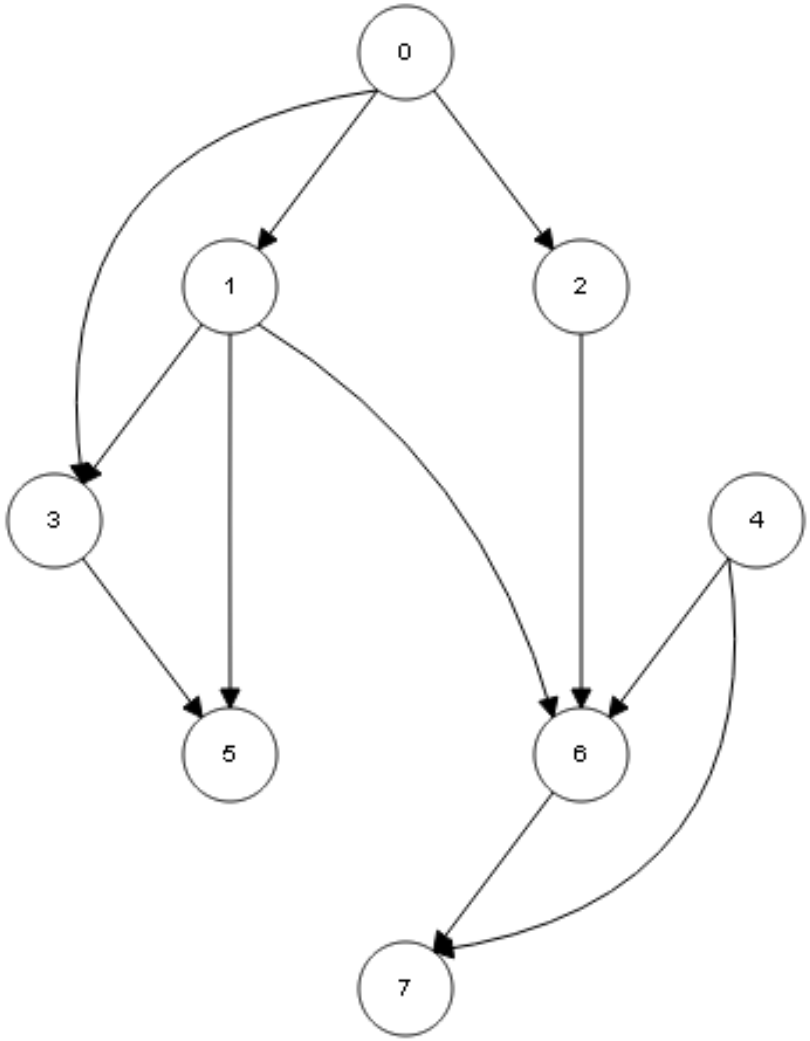


Ordenação Topológica – DFS

L 5

	0	1	2	3	4	5	6	7
c	g	g	w	g	w	b	w	w
d	1	2		3		4		
f						5		

```
c[u] ← gray
d[u] ← time ← time + 1
for each v ∈ Adj[u]
  if c[v] = white
    π[v] ← u
    visita(v)
c[u] ← black
f[u] ← time ← time + 1
insertFront(L, u)
```

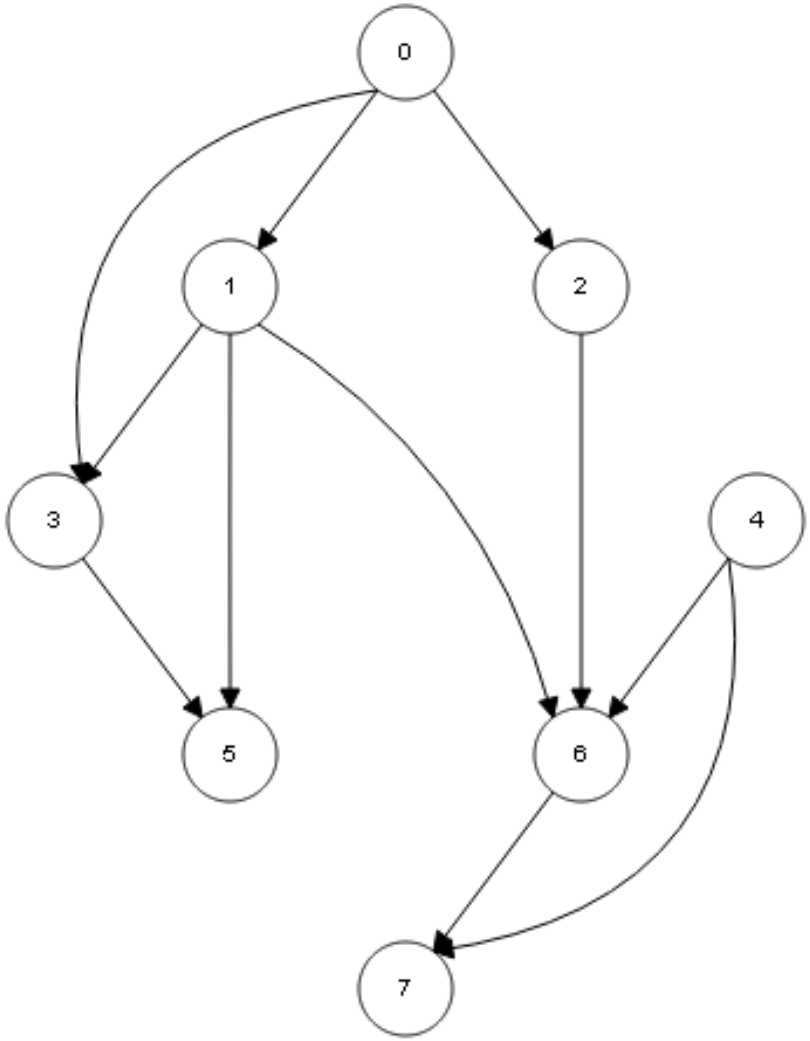


Ordenação Topológica – DFS

L | 5 |

	0	1	2	3	4	5	6	7
c	g	g	w	b	w	b	w	w
d	1	2		3		4		
f				6		5		

```
c[u] ← gray
d[u] ← time ← time + 1
for each v ∈ Adj[u]
  if c[v] = white
    π[v] ← u
    visita(v)
c[u] ← black
f[u] ← time ← time + 1
insertFront(L, u)
```

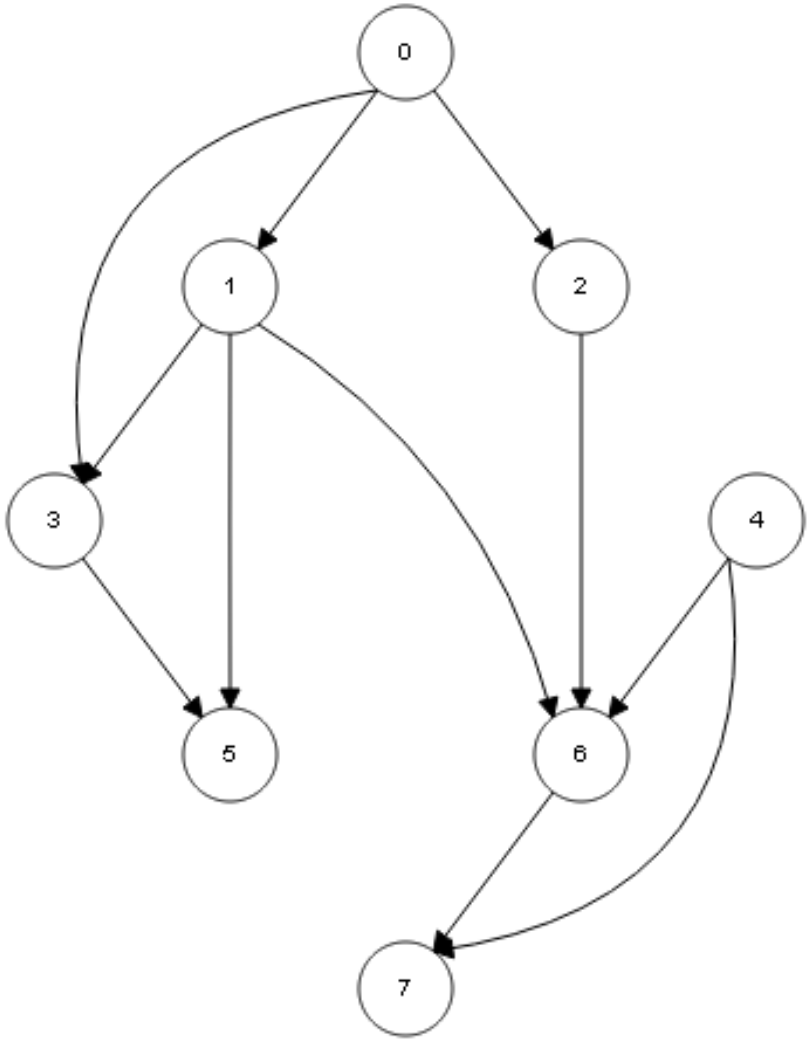


Ordenação Topológica – DFS

L | 3 | 5 |

	0	1	2	3	4	5	6	7
c	g	g	w	b	w	b	w	w
d	1	2		3		4		
f				6		5		

```
c[u] ← gray
d[u] ← time ← time + 1
for each v ∈ Adj[u]
  if c[v] = white
    π[v] ← u
    visita(v)
c[u] ← black
f[u] ← time ← time + 1
insertFront(L, u)
```

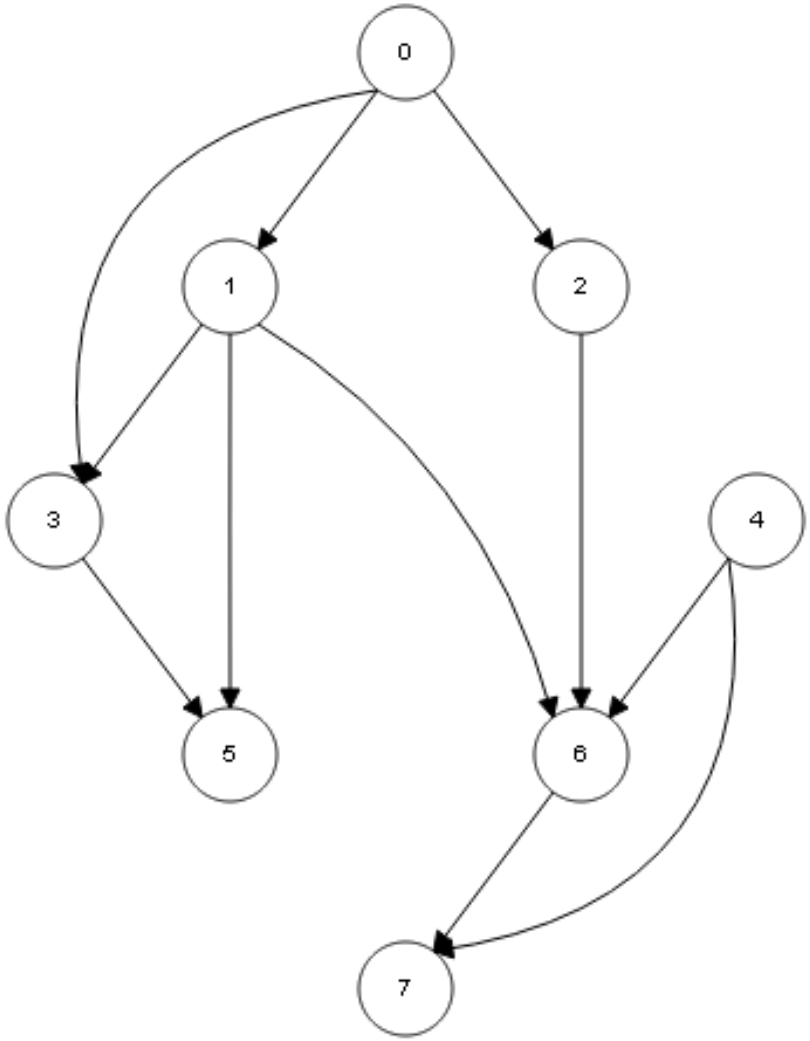


Ordenação Topológica – DFS

L	3	5

	0	1	2	3	4	5	6	7
c	g	g	w	b	w	b	g	w
d	1	2		3		4	7	
f				6		5		

```
c[u] ← gray
d[u] ← time ← time + 1
for each v ∈ Adj[u]
  if c[v] = white
    π[v] ← u
    visita(v)
c[u] ← black
f[u] ← time ← time + 1
insertFront(L, u)
```



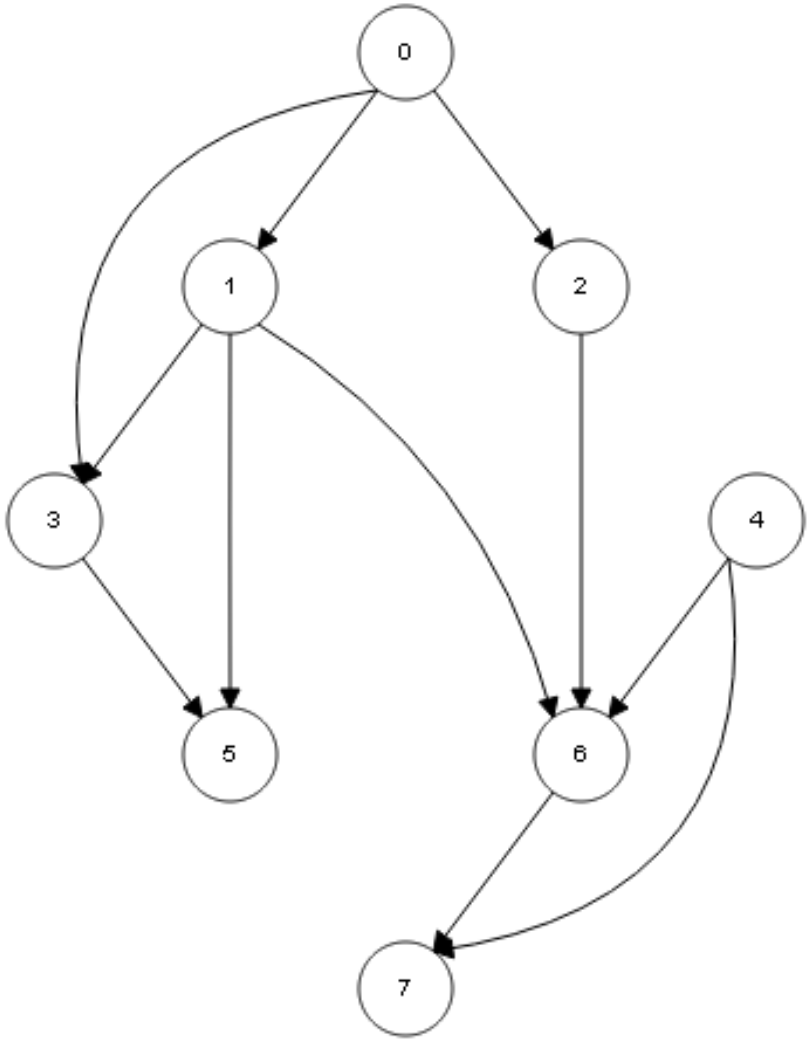
Ordenação Topológica – DFS

L

3	5

	0	1	2	3	4	5	6	7
c	g	g	w	b	w	b	g	g
d	1	2		3		4	7	8
f				6		5		

```
c[u] ← gray
d[u] ← time ← time + 1
for each v ∈ Adj[u]
  if c[v] = white
    π[v] ← u
    visita(v)
c[u] ← black
f[u] ← time ← time + 1
insertFront(L, u)
```



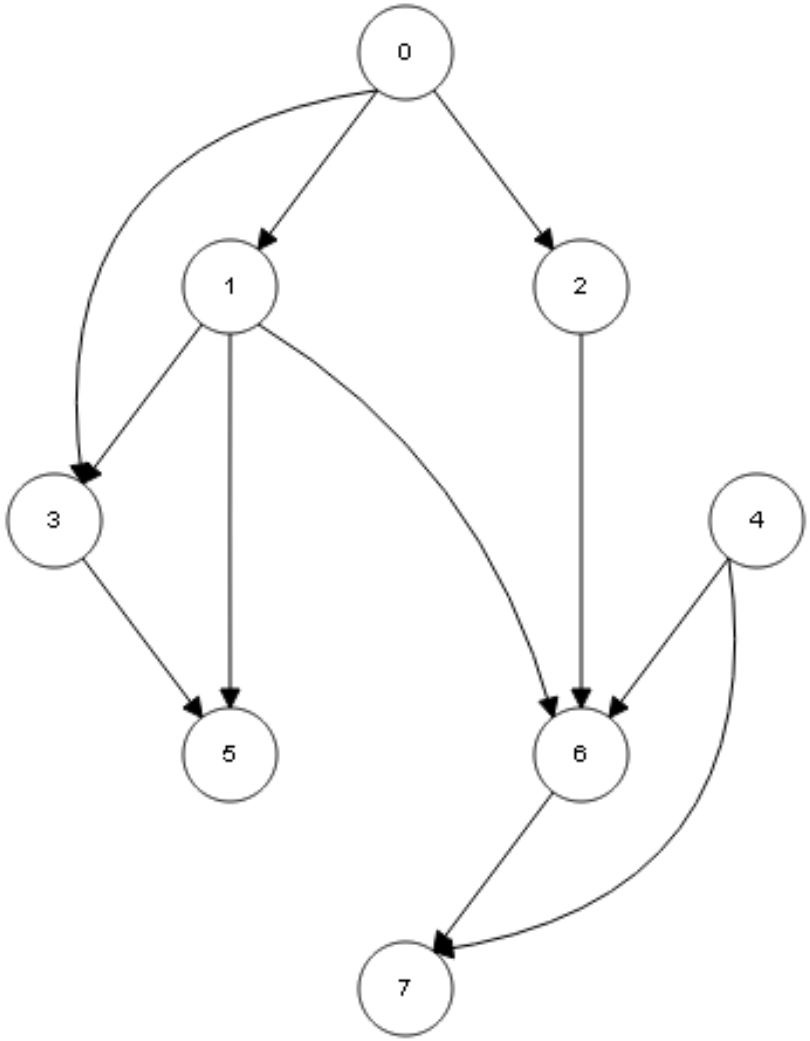
Ordenação Topológica – DFS

L

3	5

	0	1	2	3	4	5	6	7
c	g	g	w	b	w	b	g	b
d	1	2		3		4	7	8
f				6		5		9

```
c[u] ← gray
d[u] ← time ← time + 1
for each v ∈ Adj[u]
  if c[v] = white
    π[v] ← u
    visita(v)
c[u] ← black
f[u] ← time ← time + 1
insertFront(L, u)
```



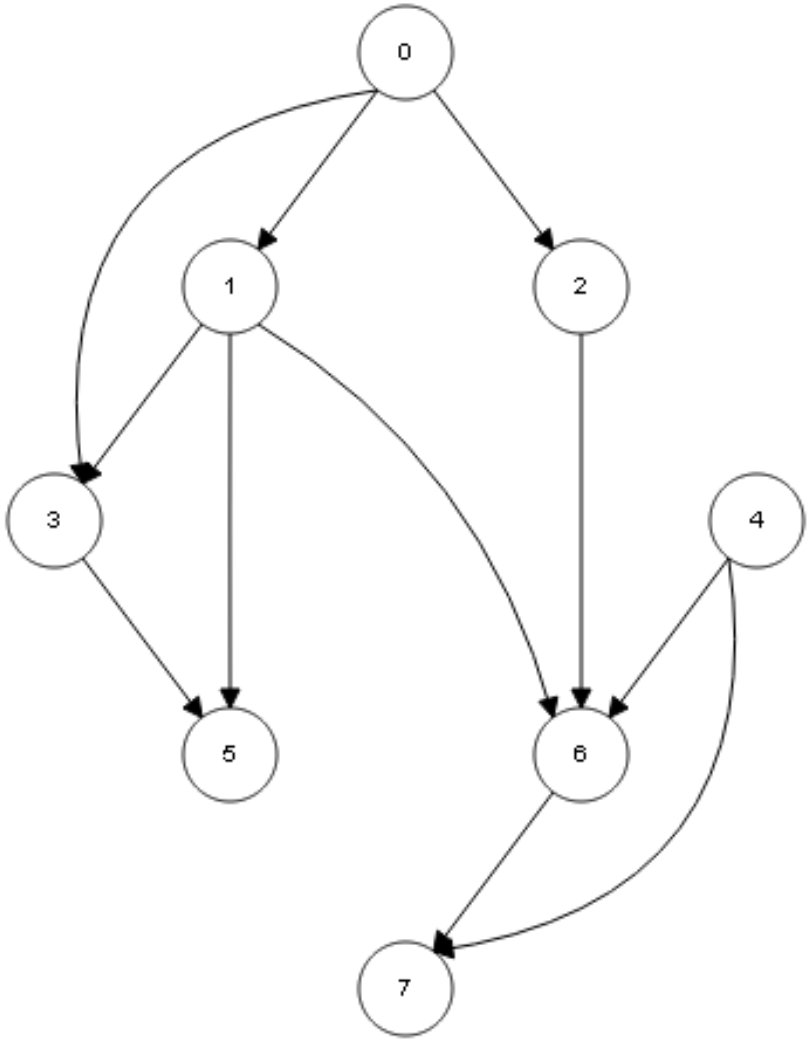
Ordenação Topológica – DFS

L

7	3	5

	0	1	2	3	4	5	6	7
c	g	g	w	b	w	b	g	b
d	1	2		3		4	7	8
f				6		5		9

```
c[u] ← gray
d[u] ← time ← time + 1
for each v ∈ Adj[u]
  if c[v] = white
    π[v] ← u
    visita(v)
c[u] ← black
f[u] ← time ← time + 1
insertFront(L, u)
```

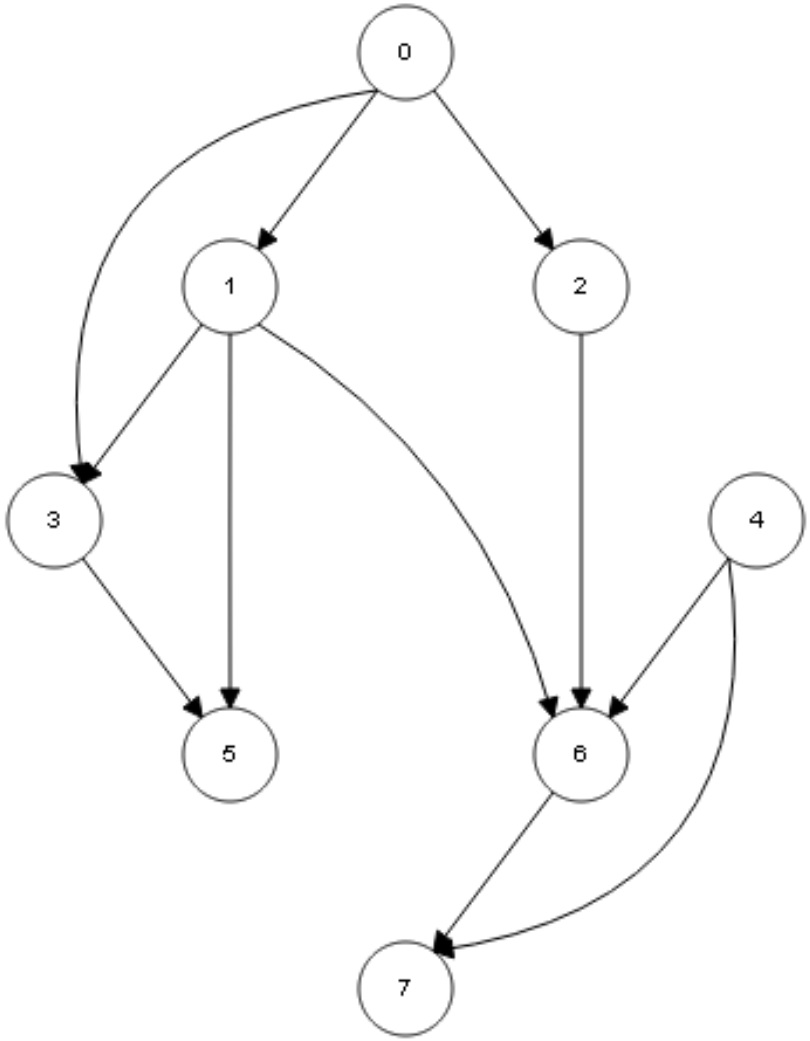


Ordenação Topológica – DFS

L	7	3	5

	0	1	2	3	4	5	6	7
c	g	g	w	b	w	b	b	b
d	1	2		3		4	7	8
f				6		5	10	9

```
c[u] ← gray
d[u] ← time ← time + 1
for each v ∈ Adj[u]
  if c[v] = white
    π[v] ← u
    visita(v)
c[u] ← black
f[u] ← time ← time + 1
insertFront(L, u)
```

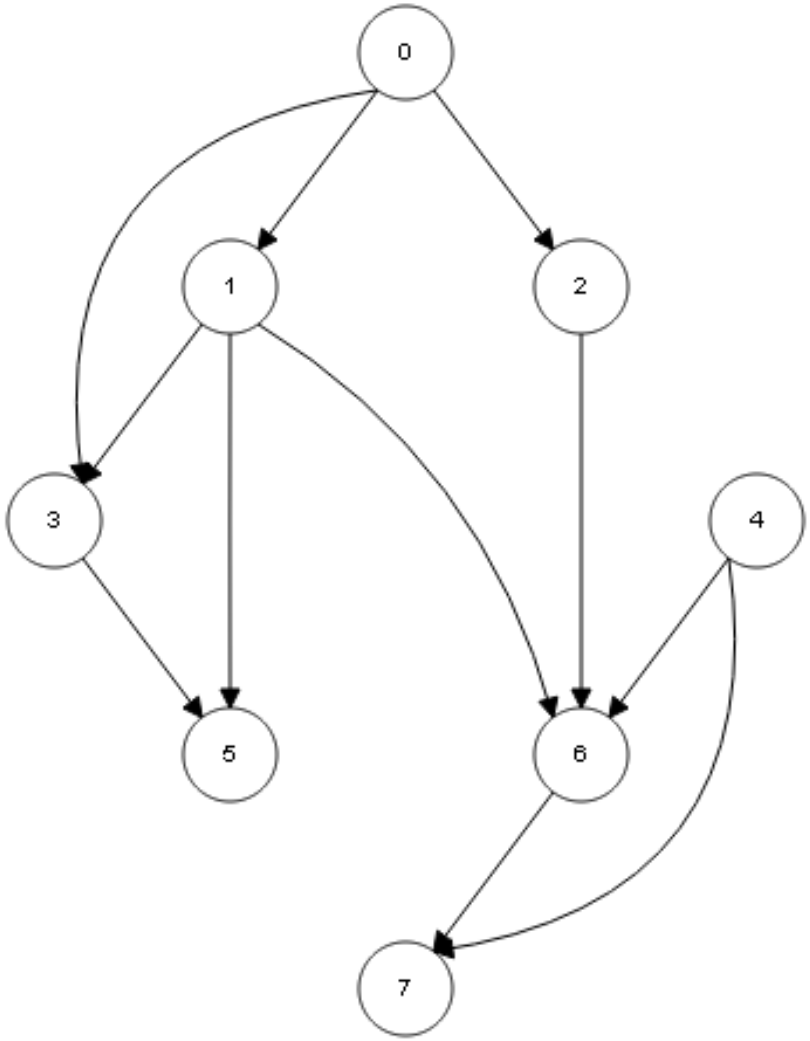


Ordenação Topológica – DFS

L	6	7	3	5

	0	1	2	3	4	5	6	7
c	g	g	w	b	w	b	b	b
d	1	2		3		4	7	8
f				6		5	10	9

```
c[u] ← gray
d[u] ← time ← time + 1
for each v ∈ Adj[u]
  if c[v] = white
    π[v] ← u
    visita(v)
c[u] ← black
f[u] ← time ← time + 1
insertFront(L, u)
```

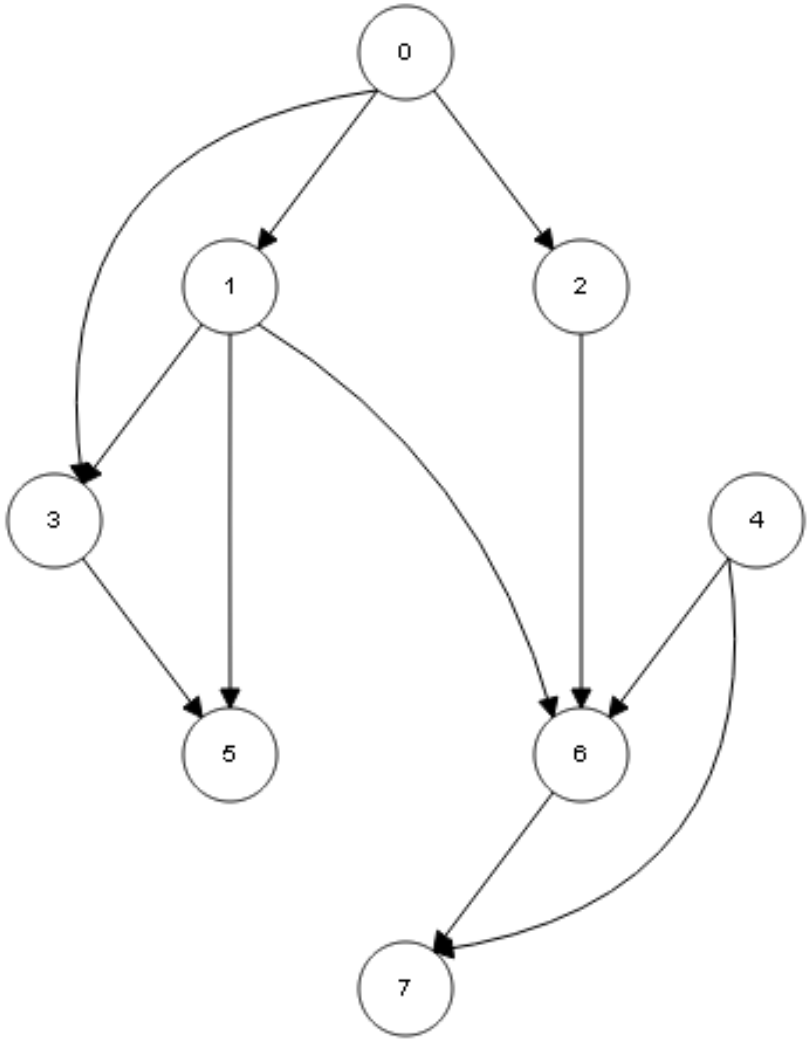


Ordenação Topológica – DFS

L	6	7	3	5

	0	1	2	3	4	5	6	7
c	g	b	w	b	w	b	b	b
d	1	2		3		4	7	8
f		11		6		5	10	9

```
c[u] ← gray
d[u] ← time ← time + 1
for each v ∈ Adj[u]
  if c[v] = white
    π[v] ← u
    visita(v)
c[u] ← black
f[u] ← time ← time + 1
insertFront(L, u)
```



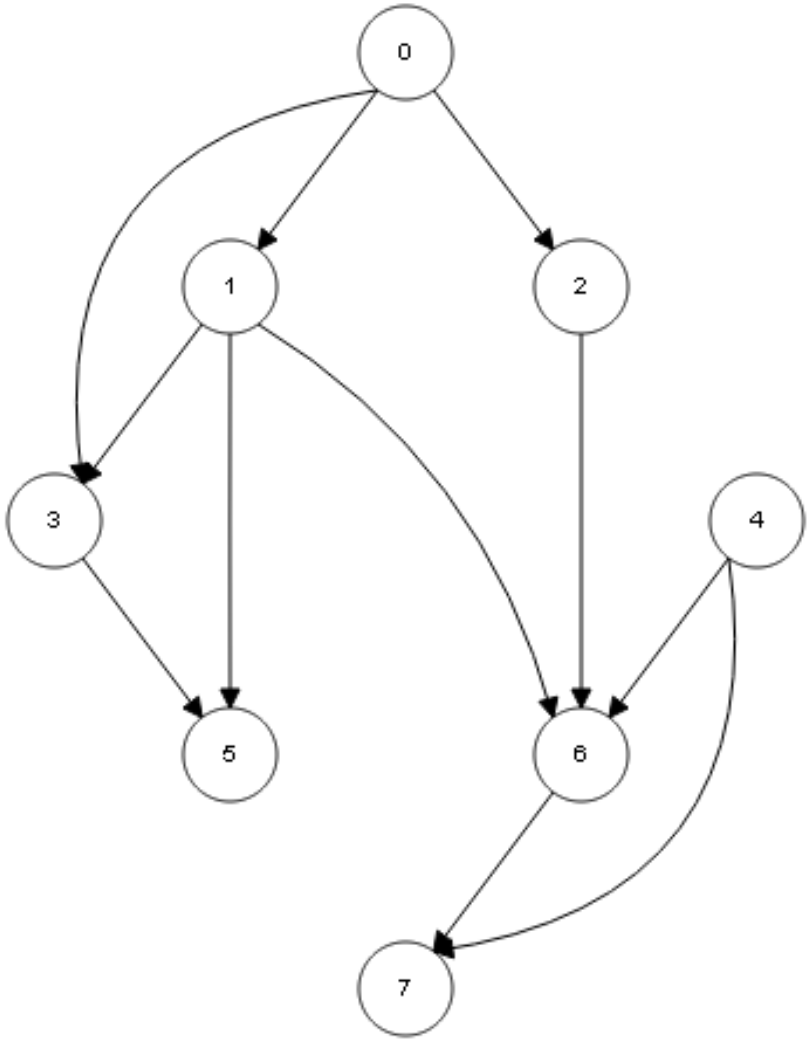
Ordenação Topológica – DFS

L

1	6	7	3	5

	0	1	2	3	4	5	6	7
c	g	b	w	b	w	b	b	b
d	1	2		3		4	7	8
f		11		6		5	10	9

```
c[u] ← gray
d[u] ← time ← time + 1
for each v ∈ Adj[u]
  if c[v] = white
    π[v] ← u
    visita(v)
c[u] ← black
f[u] ← time ← time + 1
insertFront(L, u)
```

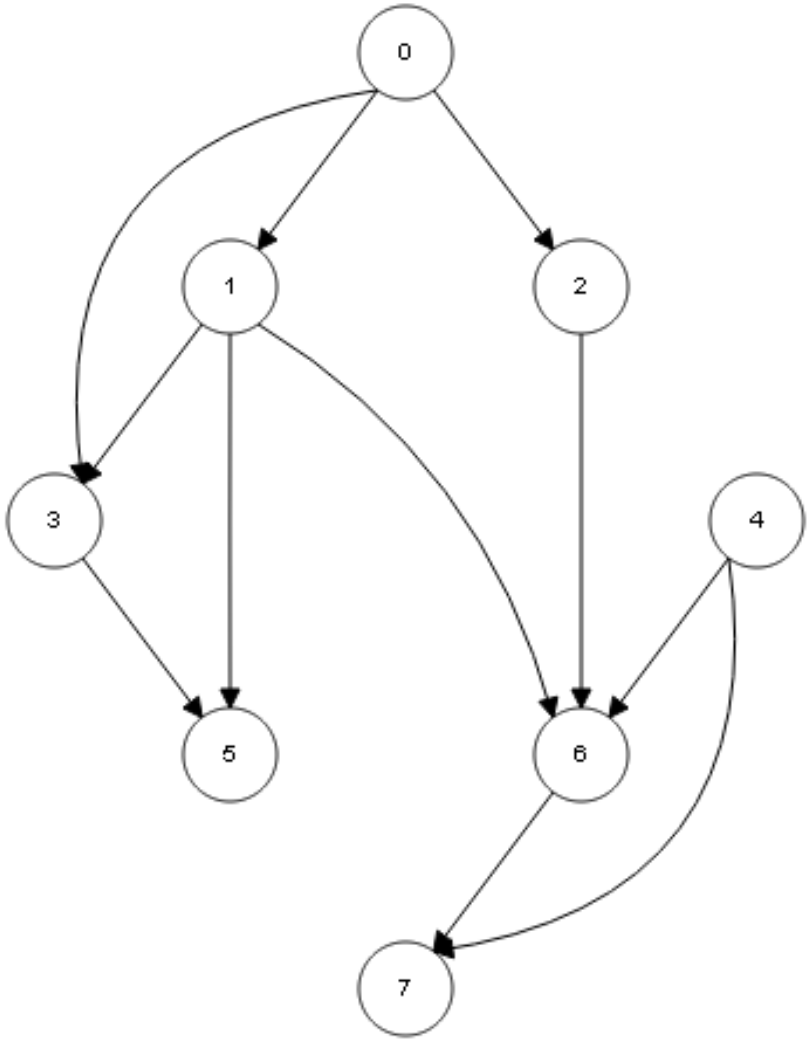


Ordenação Topológica – DFS

L	1	6	7	3	5

	0	1	2	3	4	5	6	7
c	g	b	g	b	w	b	b	b
d	1	2	12	3		4	7	8
f		11		6		5	10	9

```
c[u] ← gray
d[u] ← time ← time + 1
for each v ∈ Adj[u]
  if c[v] = white
    π[v] ← u
    visita(v)
c[u] ← black
f[u] ← time ← time + 1
insertFront(L, u)
```



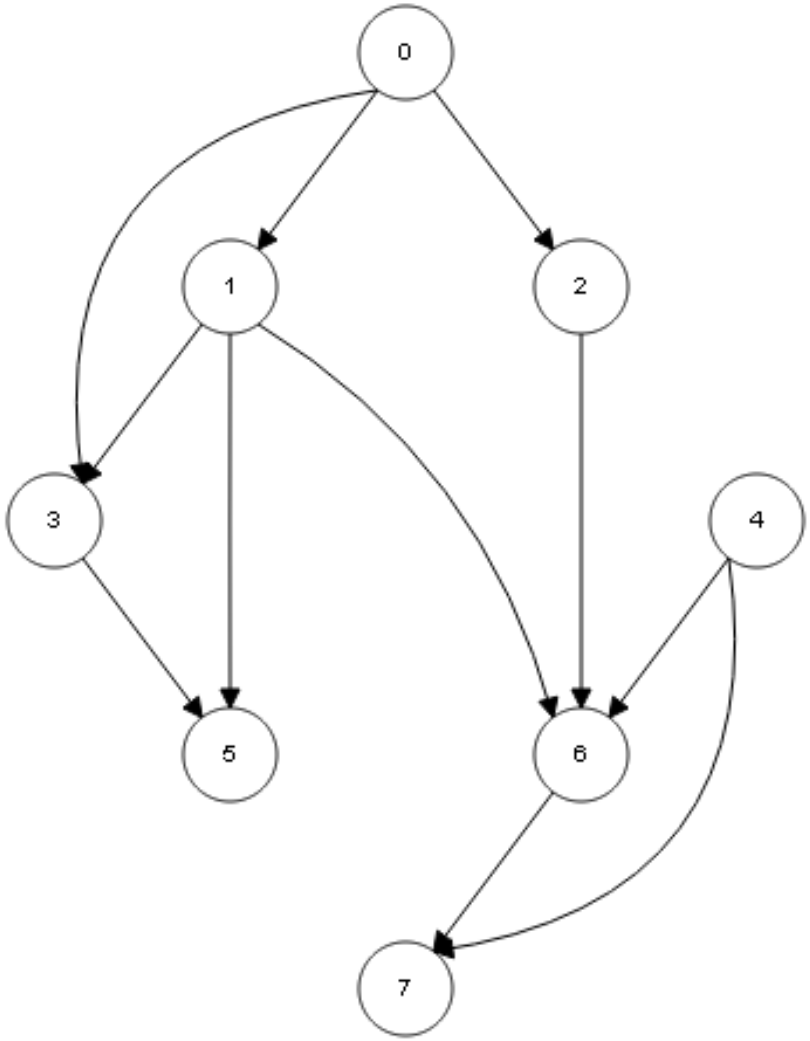
Ordenação Topológica – DFS

L

1	6	7	3	5

	0	1	2	3	4	5	6	7
c	g	b	b	b	w	b	b	b
d	1	2	12	3		4	7	8
f		11	13	6		5	10	9

```
c[u] ← gray
d[u] ← time ← time + 1
for each v ∈ Adj[u]
  if c[v] = white
    π[v] ← u
    visita(v)
c[u] ← black
f[u] ← time ← time + 1
insertFront(L, u)
```

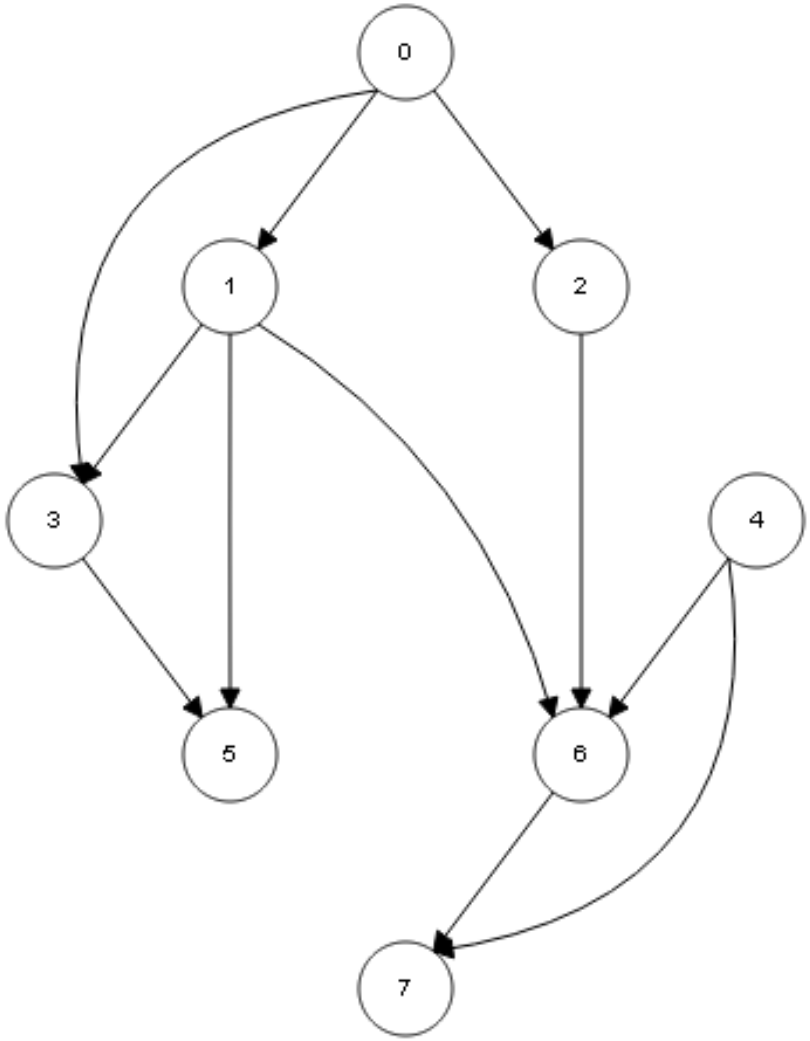


Ordenação Topológica – DFS

L	2	1	6	7	3	5

	0	1	2	3	4	5	6	7
c	g	b	b	b	w	b	b	b
d	1	2	12	3		4	7	8
f		11	13	6		5	10	9

```
c[u] ← gray
d[u] ← time ← time + 1
for each v ∈ Adj[u]
  if c[v] = white
    π[v] ← u
    visita(v)
c[u] ← black
f[u] ← time ← time + 1
insertFront(L, u)
```

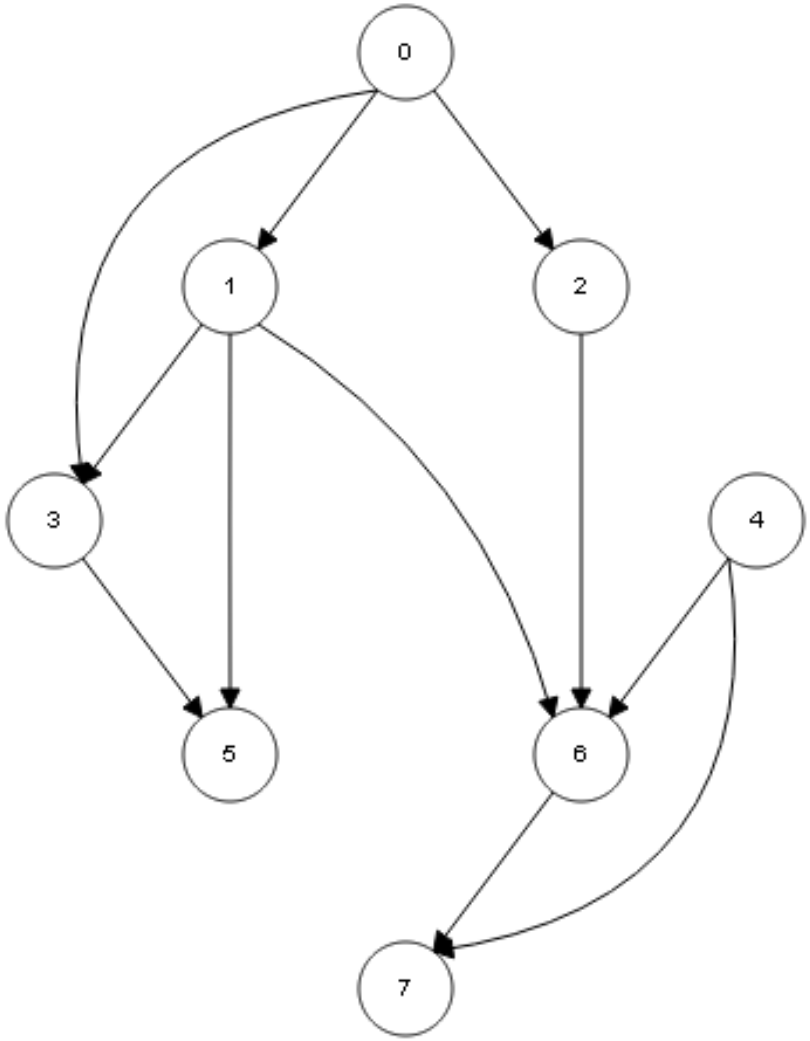


Ordenação Topológica – DFS

L	2	1	6	7	3	5

	0	1	2	3	4	5	6	7
c	b	b	b	b	w	b	b	b
d	1	2	12	3		4	7	8
f	14	11	13	6		5	10	9

```
c[u] ← gray
d[u] ← time ← time + 1
for each v ∈ Adj[u]
  if c[v] = white
    π[v] ← u
    visita(v)
c[u] ← black
f[u] ← time ← time + 1
insertFront(L, u)
```

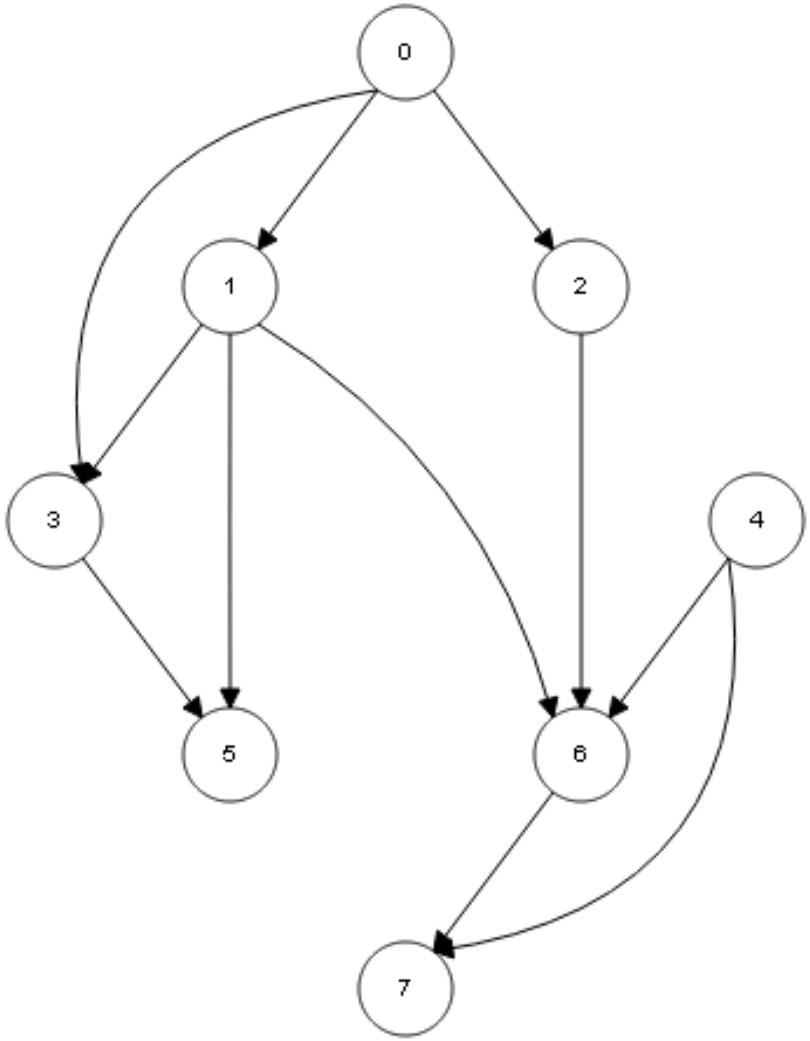


Ordenação Topológica – DFS

L	0	2	1	6	7	3	5

	0	1	2	3	4	5	6	7
c	b	b	b	b	w	b	b	b
d	1	2	12	3		4	7	8
f	14	11	13	6		5	10	9

```
c[u] ← gray
d[u] ← time ← time + 1
for each v ∈ Adj[u]
  if c[v] = white
    π[v] ← u
    visita(v)
c[u] ← black
f[u] ← time ← time + 1
insertFront(L, u)
```

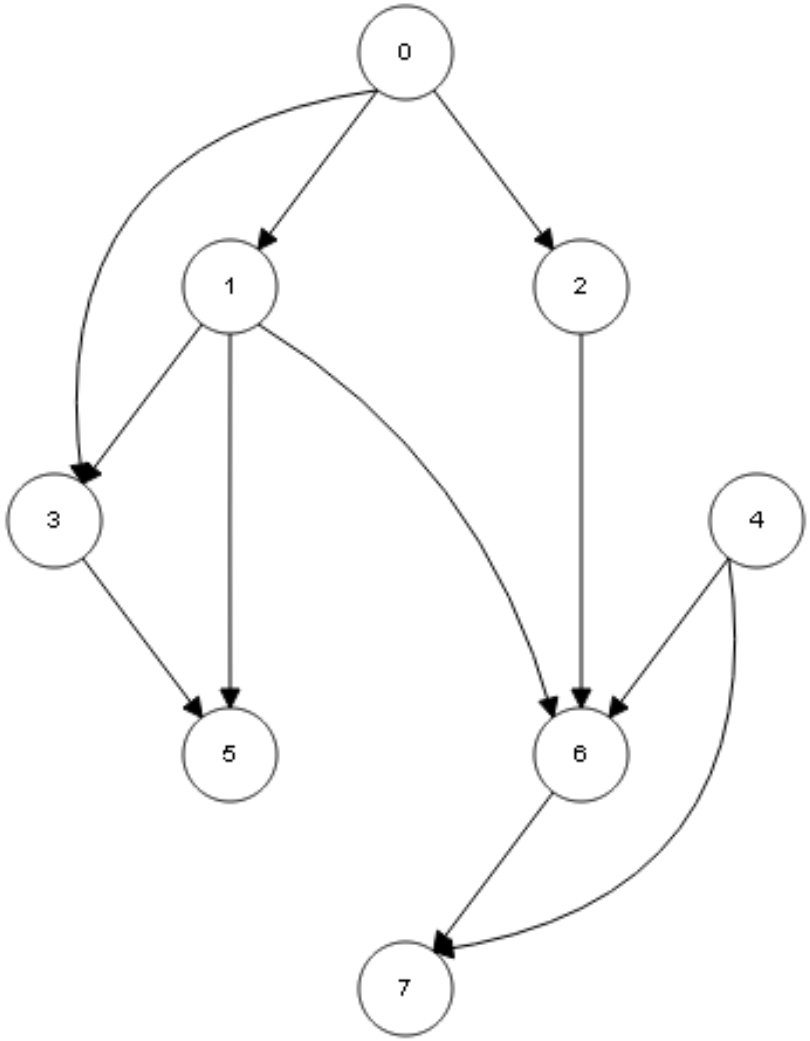


Ordenação Topológica – DFS

L	0	2	1	6	7	3	5

	0	1	2	3	4	5	6	7
c	b	b	b	b	b	b	b	b
d	1	2	12	3	15	4	7	8
f	14	11	13	6	16	5	10	9

```
c[u] ← gray
d[u] ← time ← time + 1
for each v ∈ Adj[u]
  if c[v] = white
    π[v] ← u
    visita(v)
c[u] ← black
f[u] ← time ← time + 1
insertFront(L, u)
```

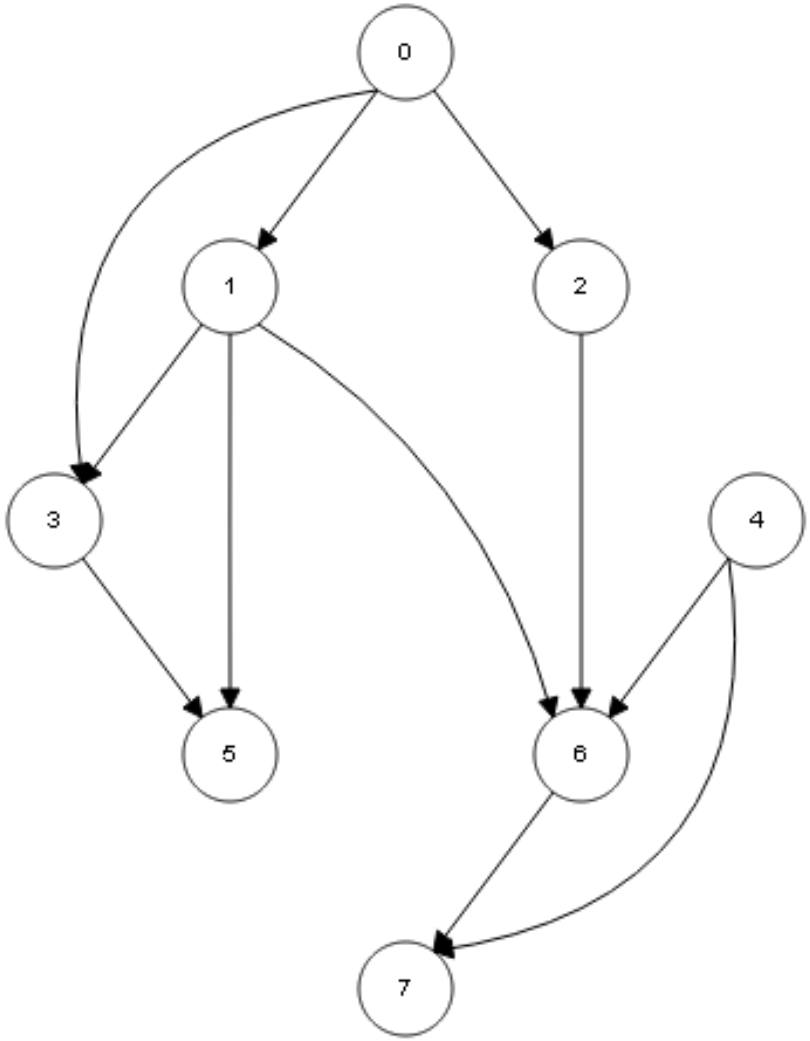


Ordenação Topológica – DFS

L	4	0	2	1	6	7	3	5

	0	1	2	3	4	5	6	7
c	b	b	b	b	b	b	b	b
d	1	2	12	3	15	4	7	8
f	14	11	13	6	16	5	10	9

```
c[u] ← gray
d[u] ← time ← time + 1
for each v ∈ Adj[u]
  if c[v] = white
    π[v] ← u
    visita(v)
c[u] ← black
f[u] ← time ← time + 1
insertFront(L, u)
```

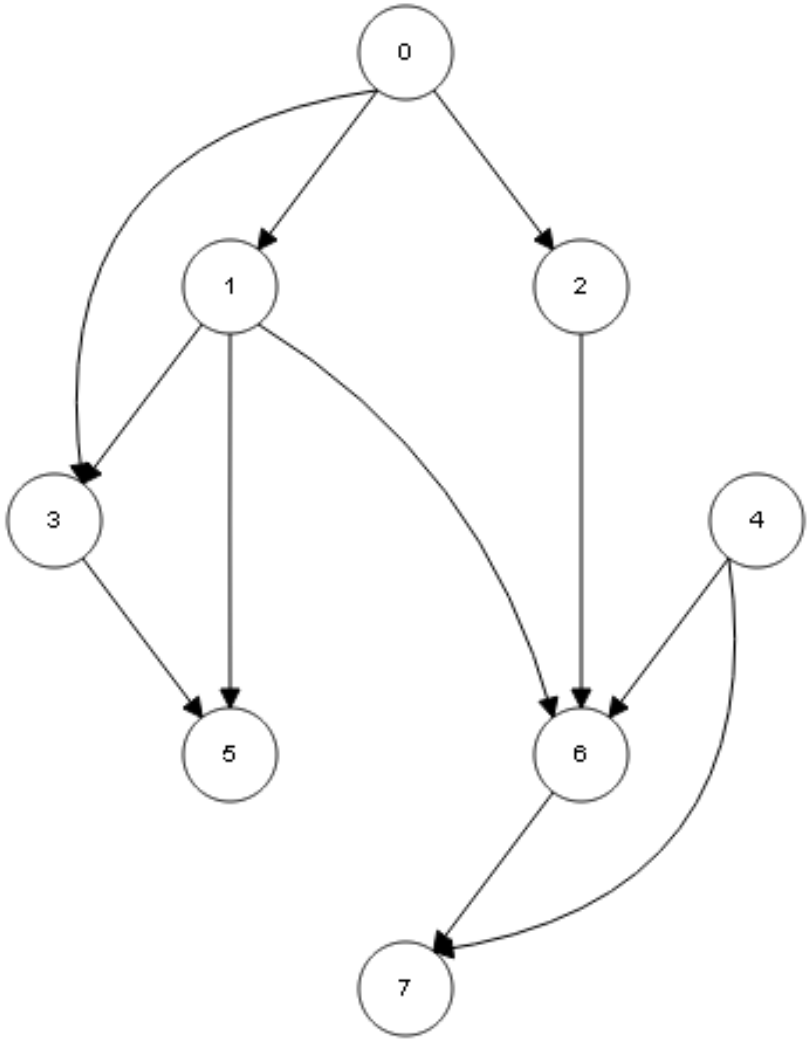


Ordenação Topológica – DFS

L	4	0	2	1	6	7	3	5

	0	1	2	3	4	5	6	7
c	b	b	b	b	b	b	b	b
d	1	2	12	3	15	4	7	8
f	14	11	13	6	16	5	10	9

```
OrdenacaoTopologicaDFS(G)
  L ← ∅
  BuscaEmProfundidade(G)
  return L
```



Ordenação Topológica (DFS) – Análise

```
OrdenacaoTopologicaDFS(G)
  L ← ∅
  BuscaEmProfundidade(G)
  return L
```

- Complexidade: **O(V + A)**

Ordenação Topológica – Aplicações

- **Dependência entre tarefas:** uma ordenação topológica é uma sequência válida de tarefas;
- **Pré-requisitos de disciplinas:** uma ordenação topológica é uma sequência válida para se cursar as disciplinas;
- **Instalação de pacotes:** uma ordenação topológica é uma sequência válida para instalação de pacotes com dependências;
- **Compilação de sistemas:** uma ordenação topológica é uma sequência válida para compilar bibliotecas com dependências;

Exercícios

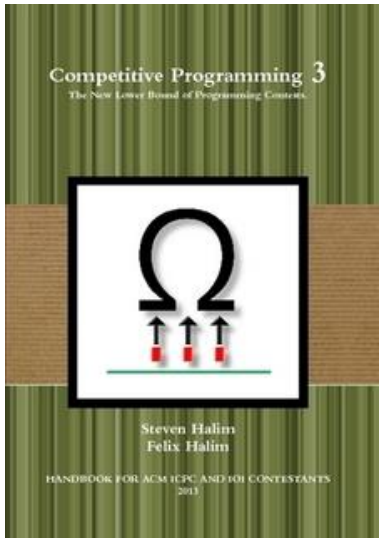
Lista de Exercícios 13 – Ordenação Topológica

http://www.inf.puc-rio.br/~elima/paa/

Leitura Complementar

- Halim e Halim. **Competitive Programming**, 3rd Edition, 2003.

- **Capítulo 4: Graph**



- Cormen, Leiserson, Rivest e Stein. **Algoritmos – Teoria e Prática**, 2ª. Edição, Editora Campus, 2002.

- **Capítulo 22: Algoritmos Elementares de Grafos**