

Programming Fundamentals

Lecture 06 – Vectors, Physics and Collision Detection

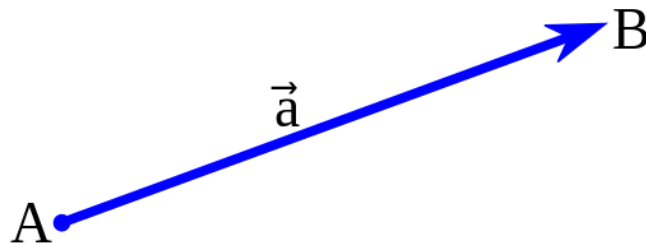
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What is a Vector?

- The word vector can mean a lot of different things...
 - Biology/epidemiology: an organism that transmits infection from one host to another;
 - Computer science: a one-dimensional array;
 - Mathematics and physics: **an entity that has both magnitude and direction.**
- A vector is typically represented as a arrow, where the direction is indicated by where the arrow is pointing, and the magnitude by the length of the arrow itself.



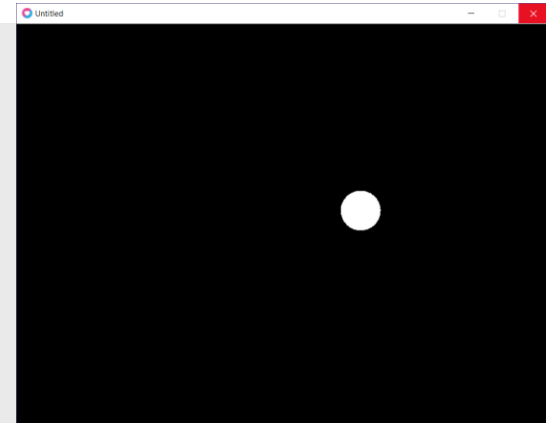
Bouncing Ball Without Vectors

```
local x = 100
local y = 100
local xspeed = 300
local yspeed = 500
local radius = 30

function love.update(dt)
    x = x + (xspeed * dt)
    y = y + (yspeed * dt)

    if (x > love.graphics.getWidth() - radius) or (x < 0 + radius) then
        xspeed = xspeed * -1;
    end
    if (y > love.graphics.getHeight() - radius) or (y < 0 + radius) then
        yspeed = yspeed * -1;
    end
end

function love.draw()
    love.graphics.circle("fill", x, y, radius, 30)
end
```

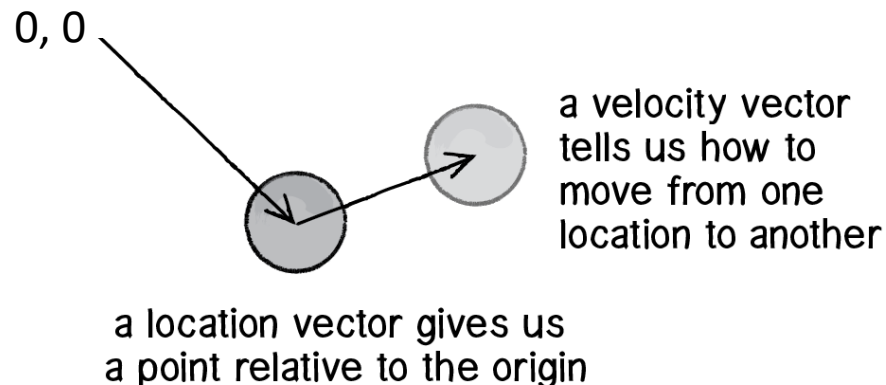


Bouncing Ball Without Vectors

- In the code, the ball has some properties that are represented by variables:

Location	x and y
Velocity	xspeed and yspeed

- Both location and velocity can be described as vectors:



Defining a Vector

- To better represent vectors in Lua, we can create a table to store both x and y:

```
vector2 = {}
```

```
function vector2.new(px, py)  
    return {x = px, y = py}  
end
```

vector2.lua

- We can save it in a separated module (file) and handle it as a new data type.

```
main.lua x vector2.lua  
1  require "vector2"  
2  
3  local position = vector2.new(100, 100)  
4  local velocity = vector2.new(300, 500)  
5  local radius = 30  
6
```

```
main.lua vector2.lua x  
1  vector2 = {}  
2  
3  function vector2.new(px, py)  
4      return {x = px, y = py}  
5  end  
6
```

What is a Table?

- A Lua table is defined by a set of **pairs key-data**, where the data is referenced by the key.
 - The key (index) can be of any type of data (number, text, ...);
 - The data can also be of any type of data (number, text, function, image);
- With Lua tables we can associate names to its elements and create structures to store related data:

```
player1 = {  
  name    = "John",  
  score   = 1000,  
  lives   = 3  
}
```

- We can access an element of a table by its name:

```
io.write(player1.score)
```

Vector Addition

- Now that we have two vector objects (location and velocity), we are ready to implement the algorithm for motion:
 - position = position + velocity
 - Code without vectors:

```
x = x + (xspeed * dt)
y = y + (yspeed * dt)
```

- However, we are not allowed to do that directly, we need a function to add two vectors:

$$\vec{w} = \vec{u} + \vec{v} = \begin{matrix} w_x = u_x + v_x \\ w_y = u_y + v_y \end{matrix}$$

```
function vector2.add(vec, inc)
  local result = vector2.new(0, 0)
  result.x = vec.x + inc.x
  result.y = vec.y + inc.y
  return result
end
```

vector2.lua

Bouncing Ball With Vectors

main.lua

```
require "vector2"

local position = vector2.new(100, 100)
local velocity = vector2.new(300, 500)
local radius = 30

function love.update(dt)
    position = vector2.add(position, vector2.mult(velocity, dt))

    if (position.x > love.graphics.getWidth() - radius) or
        (position.x < 0 + radius) then
        velocity.x = velocity.x * -1;
    end
    if (position.y > love.graphics.getHeight() - radius) or
        (position.y < 0 + radius) then
        velocity.y = velocity.y * -1;
    end
end

function love.draw()
    love.graphics.circle("fill", position.x, position.y, radius, 30)
end
```

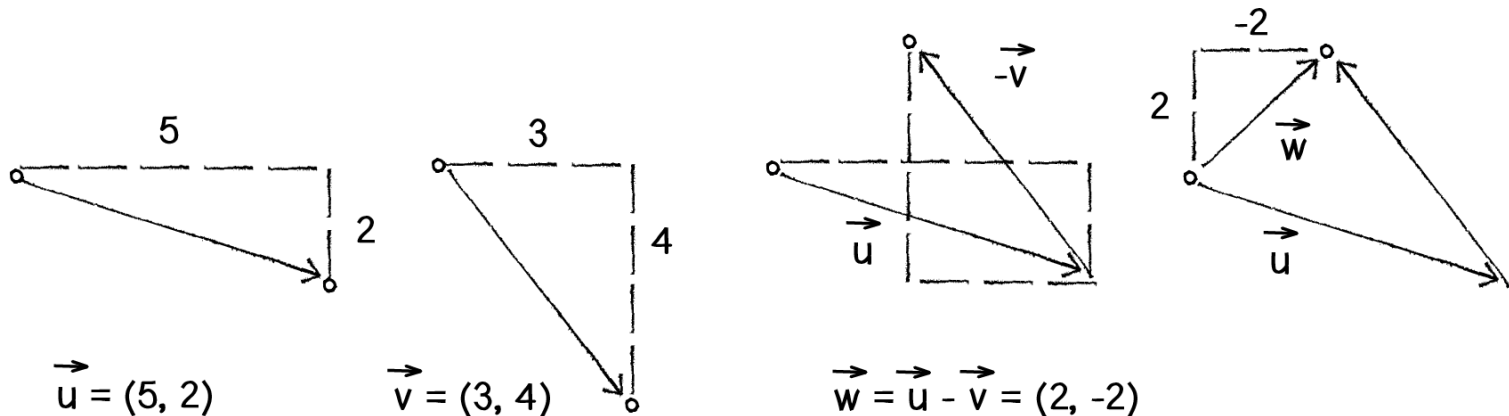

Vector Subtraction

$$\vec{w} = \vec{u} - \vec{v} = \begin{matrix} w_x = u_x - v_x \\ w_y = u_y - v_y \end{matrix}$$

- Implementation:

```
function vector2.sub(vec, dec)
  local result = vector2.new(0, 0)
  result.x = vec.x - dec.x
  result.y = vec.y - dec.y
  return result
end
```

vector2.lua



Vector Multiplication and Division

$$\vec{w} = \vec{u} * n = \begin{matrix} w_x = u_x * n \\ w_y = u_y * n \end{matrix}$$

- Multiplication implementation:

```
function vector2.mult(vec, n)
  local result = vector2.new(0, 0)
  result.x = vec.x * n
  result.y = vec.y * n
  return result
end
```

vector2.lua

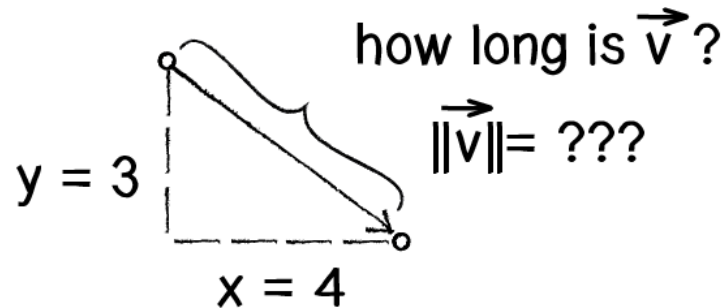
- Division implementation:

```
function vector2.div(vec, n)
  local result = vector2.new(0, 0)
  result.x = vec.x / n
  result.y = vec.y / n
  return result
end
```

vector2.lua

Vector Magnitude

$$\|\vec{v}\| = \sqrt{v_x * v_x + v_y * v_y}$$



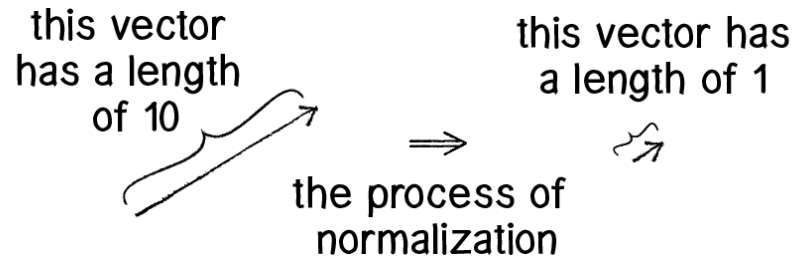
- Implementation:

```
function vector2.magnitude(vec)
  return math.sqrt(vec.x * vec.x + vec.y * vec.y)
end
```

vector2.lua

Vector Normalization

$$\hat{u} = \frac{\vec{u}}{\|\vec{u}\|}$$



- Implementation:

```
function vector2.normalize(vec)
  local m = vector2.magnitude(vec)
  if m ~= 0 then
    return vector2.div(vec, m)
  end
  return vec
end
```

vector2.lua

Vector Motion: Acceleration

- **Basic definitions:**
 - Velocity: the rate of change of location;
 - Acceleration: the rate of change of velocity;
- Acceleration affects velocity, which in turn affects location.

```
velocity = vector2.add(velocity, vector2.mult(acceleration, dt))  
position = vector2.add(position, vector2.mult(velocity, dt))
```

- Acceleration values accumulate over time in the velocity, therefore, a function to limit the velocity is required:

```
function vector2.limit(vec, max)  
    local result = vec  
    if (vector2.magnitude(vec) > max) then  
        result = vector2.normalize(vec)  
        result = vector2.mult(result, max)  
    end  
    return result  
end
```

vector2.lua

Example 1: Constant Acceleration

```
require "vector2"
```

```
main.lua
```

```
local position = vector2.new(400, 100)
```

```
local velocity = vector2.new(0, 0)
```

```
local radius = 30
```

```
local maxspeed = 500
```

```
function love.update(dt)
```

```
    local acceleration = vector2.new(-10,100)
```

```
    velocity = vector2.add(velocity, vector2.mult(acceleration, dt))
```

```
    velocity = vector2.limit(velocity, maxspeed)
```

```
    position = vector2.add(position, vector2.mult(velocity, dt))
```

```
    if (position.x > love.graphics.getWidth()) then
```

```
        position.x = 0
```

```
    elseif (position.x < 0) then
```

```
        position.x = love.graphics.getWidth()
```

```
    end
```

```
    ...
```

Example 1: Constant Acceleration

main.lua

```
...

if (position.y > love.graphics.getHeight()) then
    position.y = 0
elseif (position.y < 0) then
    position.y = love.graphics.getHeight()
end
end

function love.draw()
    love.graphics.circle("fill", position.x, position.y, radius, 30)
end
```

Example 2: Acceleration Towards the Mouse

main.lua

```
require "vector2"

local position = vector2.new(400, 100)
local velocity = vector2.new(0, 0)
local radius = 30
local maxspeed = 250

function love.update(dt)
    local mouseposition = vector2.new(love.mouse.getX(), love.mouse.getY())
    local mousedirection = vector2.sub(mouseposition, position)
    mousedirection = vector2.normalize(mousedirection)

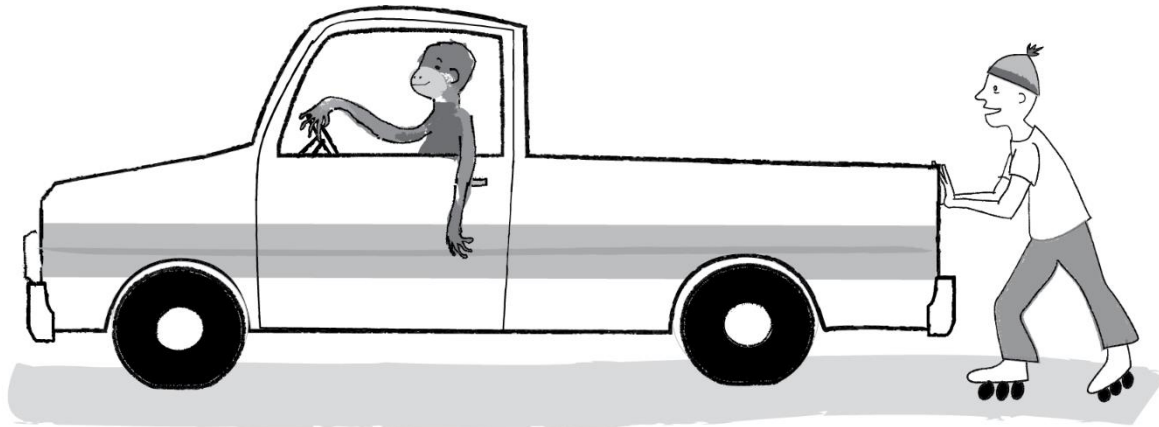
    local acceleration = vector2.mult(mousedirection, 700)

    velocity = vector2.add(velocity, vector2.mult(acceleration, dt))
    velocity = vector2.limit(velocity, maxspeed)
    position = vector2.add(position, vector2.mult(velocity, dt))
end

function love.draw()
    love.graphics.circle("fill", position.x, position.y, radius, 30)
end
```


Forces

- A force is a vector that causes an object with mass to accelerate.
- **Newton's Laws of Motion:**
 1. An object at rest stays at rest and an object in motion stays in motion.
 2. Force equals mass times acceleration.
 3. For every action there is an equal and opposite reaction.



Apply Force

- Implementation:

vector2.lua

```
function vector2.applyForce(force, mass, acceleration)
    local f = vector2.div(force, mass)
    return vector2.add(acceleration, f)
end
```

main.lua

...

```
local mass = 1
```

...

Example: Gravity and Wind Forces

```
require "vector2"
```

```
local position = vector2.new(400, 100)  
local velocity = vector2.new(0, 0)  
local radius = 30  
local maxspeed = 5  
local mass = 1
```

```
local wind = vector2.new(100, 0)  
local gravity = vector2.new(0, 500)
```

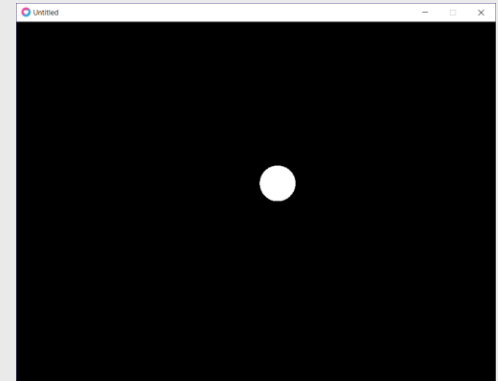
```
function love.update(dt)
```

```
    local acceleration = vector2.new(0, 0)  
    acceleration = vector2.applyForce(wind, mass, acceleration)  
    acceleration = vector2.applyForce(gravity, mass, acceleration)
```

```
    velocity = vector2.add(velocity, vector2.mult(acceleration, dt))  
    position = vector2.add(position, vector2.mult(velocity, dt))
```

```
    ...
```

main.lua



Example: Gravity and Wind Forces

main.lua

```
...

if (position.x > love.graphics.getWidth() - radius) then
    velocity.x = velocity.x * -1
elseif (position.x < radius) then
    velocity.x = velocity.x * -1
end

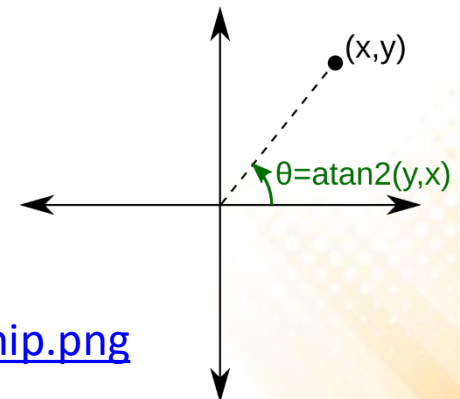
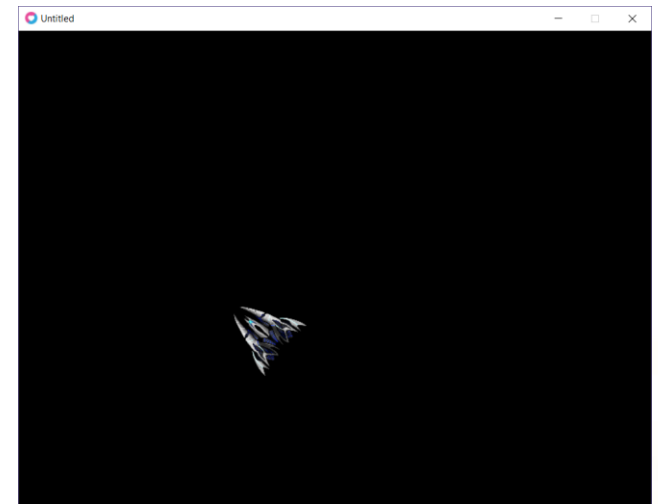
if (position.y > love.graphics.getHeight() - radius) then
    velocity.y = velocity.y * -1
end
end

function love.draw()
    love.graphics.circle("fill", position.x, position.y, radius, 30)
end
```

Exercise 1

1) Write the code to accelerate a spaceship towards the mouse position.

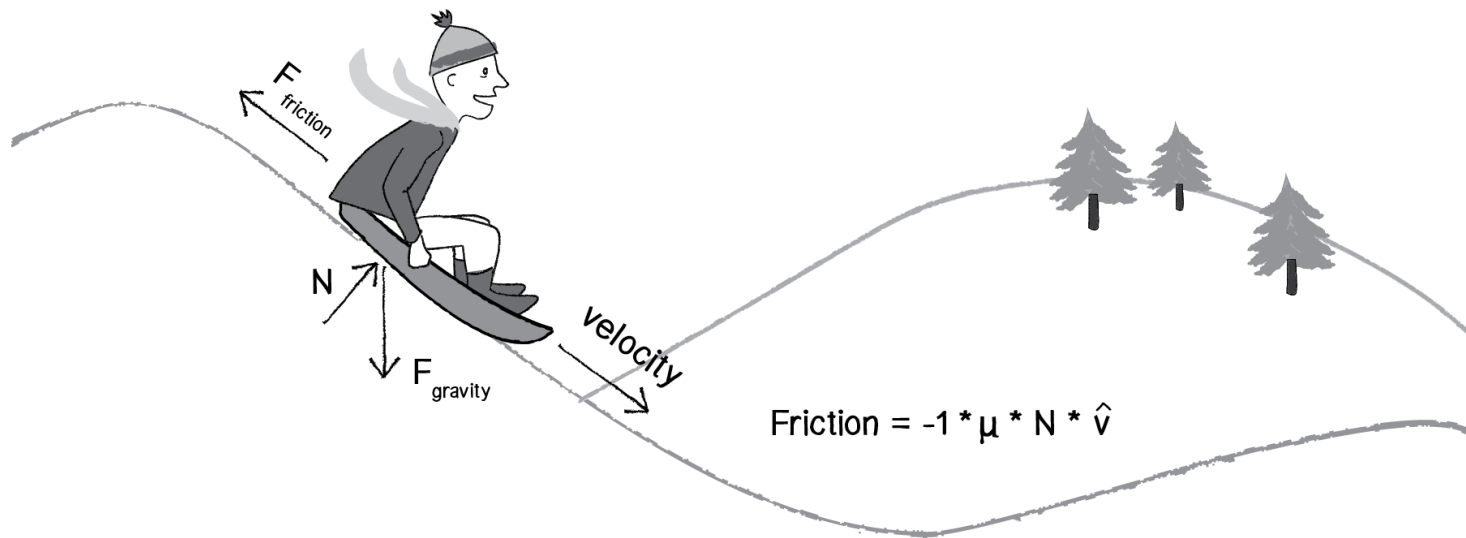
- You need to use the `applyForce` function to move the spaceship.
- Hint:
 - First, you need to calculate a vector that points from the spaceship to the mouse location (direction).
- The spaceship must rotate towards the direction of its movement.
- Hint:
 - Use the `math.atan2` function to calculate the arc tangent value from the y and x variables of the direction of the velocity.



Spaceship: <http://www.inf.puc-rio.br/~elima/gameprog/spaceship.png>

Friction

- Friction is a dissipative force, which is a type of force where the total energy of the system decreases when an object is in motion.



μ = coefficient of friction (strength of a friction force for a particular surface)

N = normal force (the force perpendicular to the object's motion along a surface)

\hat{v} = velocity unit vector

Example: Friction Force

main.lua

```
require "vector2"

local position = vector2.new(400, 100)
local velocity = vector2.new(0, 0)
local radius = 30
local maxspeed = 5
local mass = 1

local frictioncoefficient = 100
local wind = vector2.new(100,0)
local gravity = vector2.new(0, 500)

function love.update(dt)
    local friction = vector2.mult(velocity, -1)
    friction = vector2.normalize(friction)
    friction = vector2.mult(friction, frictioncoefficient)

    local acceleration = vector2.new(0, 0)
    acceleration = vector2.applyForce(friction, mass, acceleration)
    acceleration = vector2.applyForce(wind, mass, acceleration)
    acceleration = vector2.applyForce(gravity, mass, acceleration)
    ...
end
```

Example: Friction Force

main.lua

...

```
velocity = vector2.add(velocity, vector2.mult(acceleration, dt))  
position = vector2.add(position, vector2.mult(velocity, dt))
```

```
if (position.x > love.graphics.getWidth() - radius) then  
    position.x = love.graphics.getWidth() - radius  
    velocity.x = velocity.x * -1
```

```
elseif (position.x < radius) then  
    position.x = radius  
    velocity.x = velocity.x * -1
```

```
end
```

```
if (position.y > love.graphics.getHeight() - radius) then  
    position.y = love.graphics.getHeight() - radius  
    velocity.y = velocity.y * -1
```

```
end
```

```
end
```

```
function love.draw()
```

```
    love.graphics.circle("fill", position.x, position.y, radius, 30)
```

```
end
```


Example: Player Movement and Jump

```
require "vector2"
```

```
main.lua
```

```
local position = vector2.new(400, 100)
```

```
local velocity = vector2.new(0, 0)
```

```
local playerwidth = 30
```

```
local playerheight = 60
```

```
local maxspeed = 400
```

```
local mass = 1
```

```
local frictioncoefficient = 300
```

```
local onGround = false
```

```
function love.update(dt)
```

```
    local acceleration = vector2.new(0, 0)
```

```
    local gravity = vector2.new(0, 500)
```

```
    acceleration = vector2.applyForce(gravity, mass, acceleration)
```

```
    if (onGround) then
```

```
        local friction = vector2.mult(velocity, -1)
```

```
        friction = vector2.normalize(friction)
```

```
        friction = vector2.mult(friction, frictioncoefficient)
```

```
        acceleration = vector2.applyForce(friction, mass, acceleration)
```

```
        ...
```

Example: Player Movement and Jump

main.lua

```
...

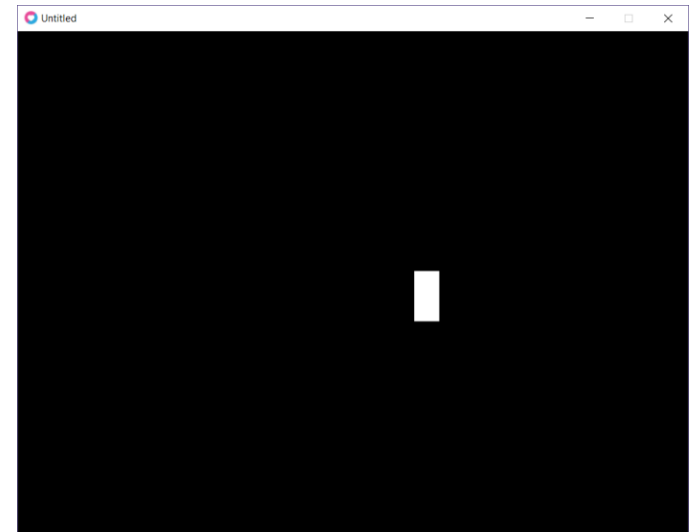
if love.keyboard.isDown("right") then
    local move = vector2.new(500, 0)
    acceleration = vector2.applyForce(move, mass, acceleration)
end
if love.keyboard.isDown("left") then
    local move = vector2.new(-500, 0)
    acceleration = vector2.applyForce(move, mass, acceleration)
end
if love.keyboard.isDown("up") then
    local jump = vector2.new(0, -50000)
    acceleration = vector2.applyForce(jump, mass, acceleration)
    onGround = false
end
end

velocity = vector2.add(velocity, vector2.mult(acceleration, dt))
velocity = vector2.limit(velocity, maxspeed)
position = vector2.add(position, vector2.mult(velocity, dt))

...
```

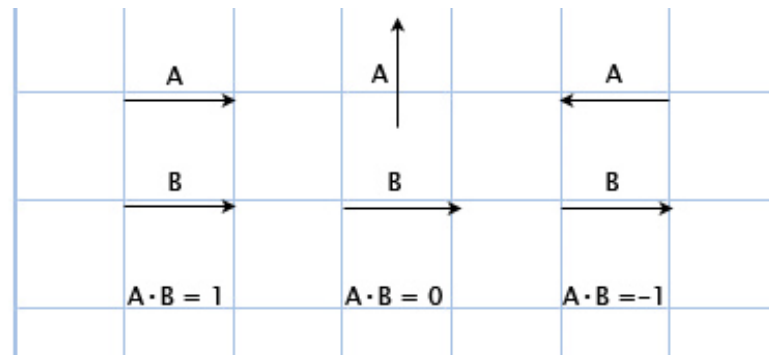

Exercise 2

- 2) Modify the code of the player movement and jump example to allow the player to double jump.
- The player must be able to jump only two times.
 - After double jumping, a new jump can only be performed when the player touches the ground.
 - Hint:
 - Use the callback `love.keyreleased(key, scancode)` to detect when the player releases the jump key, which indicates that he/she can jump again (if it was the first jump).

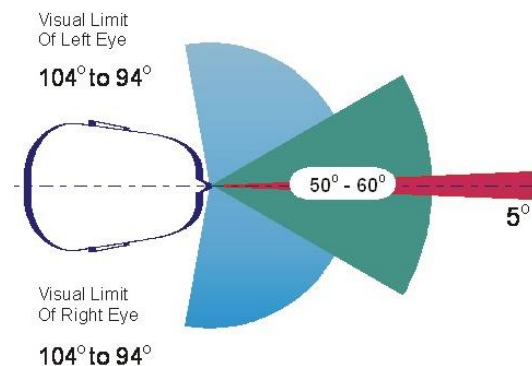


Dot Product

$$A \cdot B = (A_x, A_y) \cdot (B_x, B_y) = A_x B_x + A_y B_y$$



- The dot product has several applications in game programming.
 - Example: sight/view angle



Example: View Angle and Modularization

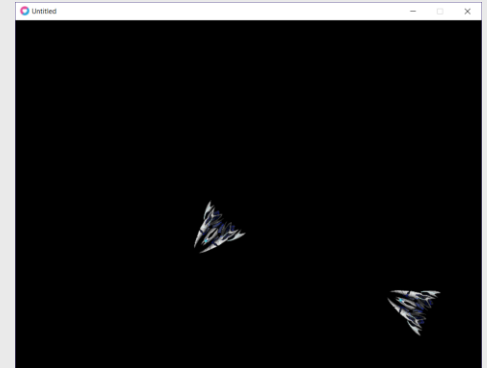
```
require "vector2"
```

```
local playerposition = vector2.new(400, 300)
local playervelocity = vector2.new(0, 0)
local playerrotation = 0
local playermaxspeed = 100
local playermass = 1
local playerimage
```

```
function LoadPlayer()
    playerimage = love.graphics.newImage("spaceship.png")
end
```

```
function UpdatePlayer(dt)
    local acceleration = vector2.new(0, 0)
    local mouseposition = vector2.new(love.mouse.getX(),
                                      love.mouse.getY())
    local mousedirection = vector2.sub(mouseposition, playerposition)
    local velocitydirection = vector2.normalize(playervelocity)
    mousedirection = vector2.normalize(mousedirection)
    ...
end
```

player.lua



Example: View Angle and Modularization

player.lua

```
...

playerrotation = math.atan2(mousedirection.y, mousedirection.x)
if (love.mouse.isDown(1)) then
    local engineForce = vector2.mult(mousedirection, 100)
    acceleration = vector2.applyForce(engineForce, playermass,
                                      acceleration)

end
playervelocity = vector2.add(playervelocity,
                             vector2.mult(acceleration, dt))
playervelocity = vector2.limit(playervelocity, playermaxspeed)
playerposition = vector2.add(playerposition,
                             vector2.mult(playervelocity, dt))
end
function DrawPlayer()
    love.graphics.draw(playerimage, playerposition.x, playerposition.y,
                      playerrotation, 0.3, 0.3, playerimage:getWidth() / 2,
                      playerimage:getHeight() / 2)
end
function GetPlayerPosition()
    return playerposition
end
```

Example: View Angle and Modularization

enemy.lua

```
require "vector2"

local enemyposition = vector2.new(700, 500)
local enemyvelocity = vector2.new(0, 0)
local enemydirection = vector2.new(-1, 0)
local enemyrotation = math.atan2(enemydirection.y, enemydirection.x)
local enemymaxspeed = 100
local enemymass = 1
local enemyviewangle = 30
local enemychasing = false
local enemyimage

function LoadEnemy()
    enemyimage = love.graphics.newImage("spaceship.png")
end

function UpdateEnemy(dt, pposition)
    local acceleration = vector2.new(0, 0)

    if (CanSee(pposition)) then
        enemychasing = true
    end
    ...
```


Example: View Angle and Modularization

...

player.lua

```
function CanSee(pposition)
    local pdirection = vector2.normalize(vector2.sub(pposition,
                                                    enemyposition))

    local angle = math.acos(vector2.dot(enemydirection, pdirection))
    if (math.deg(angle) < 30) then
        return true
    end
    return false
end
```

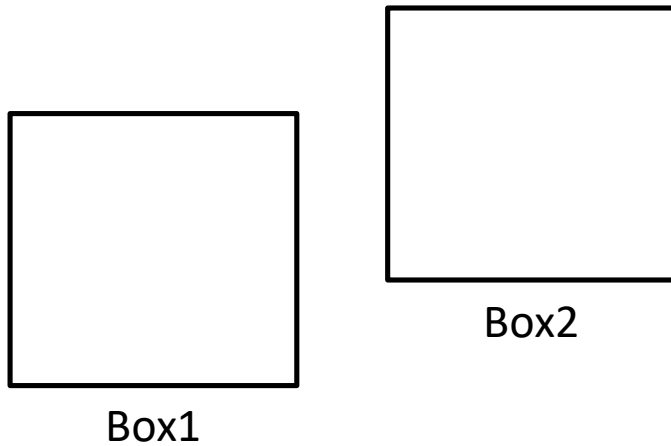
Example: View Angle and Modularization

```
require "player"  
require "enemy"  
  
function love.load()  
    LoadPlayer()  
    LoadEnemy()  
end  
  
function love.update(dt)  
    UpdatePlayer(dt)  
    UpdateEnemy(dt, GetPlayerPosition())  
end  
  
function love.draw()  
    DrawPlayer()  
    DrawEnemy()  
end
```

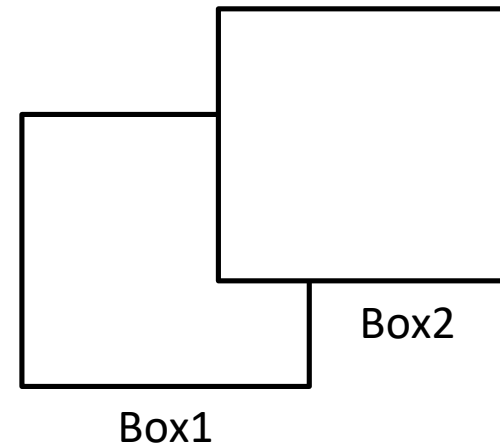
main.lua

Collision Detection

No Collision

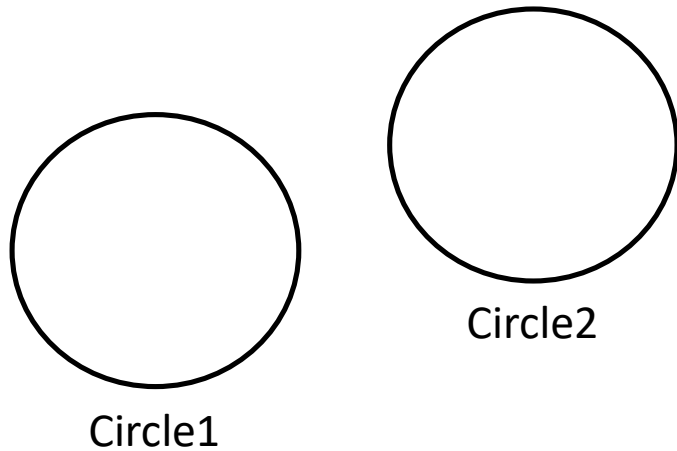


Collision

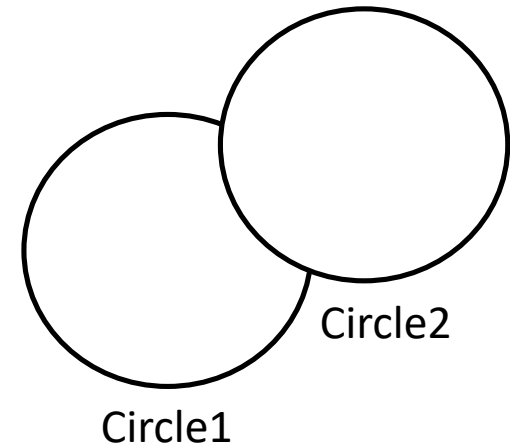


Collision Detection

No Collision



Collision



Collision Detection (Box)

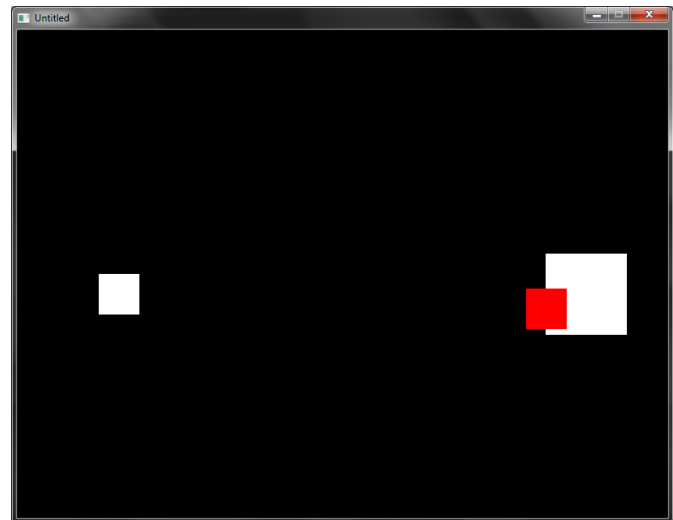
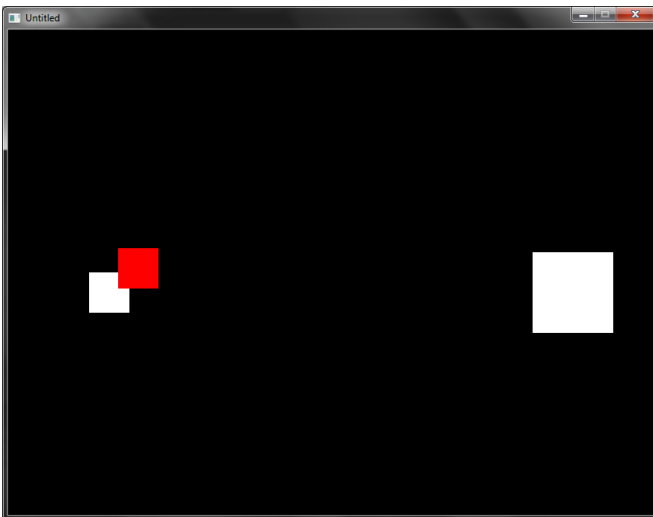
```
function love.load()
  player1 = {
    x = 390,
    y = 300,
    width = 50,
    height = 50,
    collided = false
  }
  box1 = {
    x = 100,
    y = 300,
    width = 50,
    height = 50
  }
  box2 = {
    x = 650,
    y = 275,
    width = 100,
    height = 100
  }
end
```

```
function CheckBoxCollision(x1,y1,w1,h1,x2,y2,w2,h2)
    return x1 < x2+w2 and x2 < x1+w1 and y1 < y2+h2 and y2 < y1+h1
end
```

```
function love.update(dt)
    if love.keyboard.isDown("left") then
        player1.x = player1.x - (120 * dt)
    end
    if love.keyboard.isDown("right") then
        player1.x = player1.x + (120 * dt)
    end
    if love.keyboard.isDown("up") then
        player1.y = player1.y - (120 * dt)
    end
    if love.keyboard.isDown("down") then
        player1.y = player1.y + (120 * dt)
    end

    if CheckBoxCollision(player1.x, player1.y, player1.width,
        player1.height, box1.x, box1.y, box1.width, box1.height) or
        CheckBoxCollision(player1.x, player1.y, player1.width,
        player1.height, box2.x, box2.y, box2.width, box2.height) then
        player1.collided = true
    else
        player1.collided = false
    end
end
```

```
function love.draw()  
  love.graphics.setColor(255,255,255)  
  love.graphics.rectangle("fill", box1.x, box1.y,  
                           box1.width, box1.height)  
  love.graphics.rectangle("fill", box2.x, box2.y,  
                           box2.width, box2.height)  
  
  if player1.collided == true then  
    love.graphics.setColor(255,0,0)  
  end  
  
  love.graphics.rectangle("fill", player1.x, player1.y,  
                           player1.width, player1.height)  
end
```



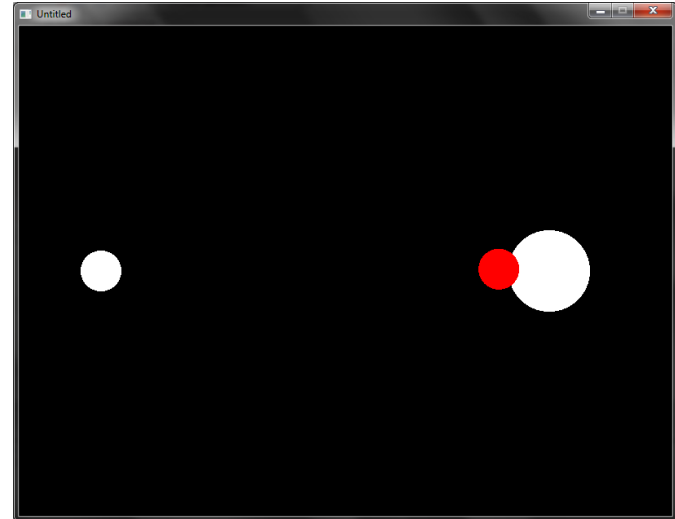
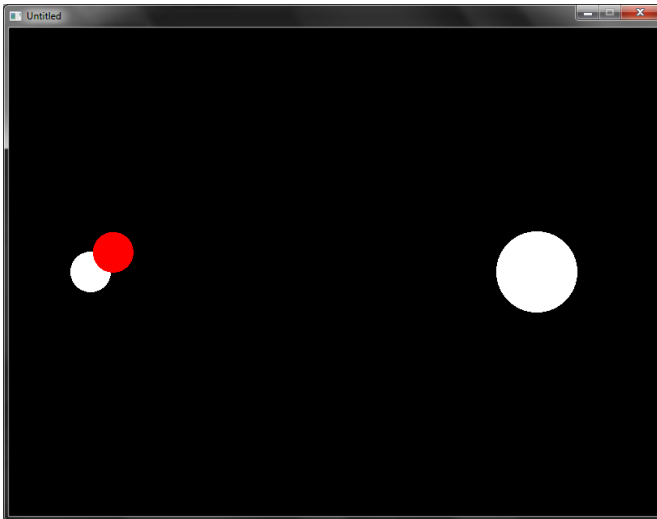
Collision Detection (Circle)

```
function love.load()
  player1 = {
    x = 390,
    y = 300,
    radius = 25,
    collided = false
  }
  circle1 = {
    x = 100,
    y = 300,
    radius = 25
  }
  circle2 = {
    x = 650,
    y = 300,
    radius = 50
  }
end
```

```
function CheckCircularCollision(ax, ay, ar, bx, by, br)
    local dx = bx - ax
    local dy = by - ay
    local dist = math.sqrt(dx * dx + dy * dy)
    return dist < ar + br
end

function love.update(dt)
    if love.keyboard.isDown("left") then
        player1.x = player1.x - (120 * dt)
    end
    if love.keyboard.isDown("right") then
        player1.x = player1.x + (120 * dt)
    end
    if love.keyboard.isDown("up") then
        player1.y = player1.y - (120 * dt)
    end
    if love.keyboard.isDown("down") then
        player1.y = player1.y + (120 * dt)
    end
    if CheckCircularCollision(player1.x, player1.y, player1.radius,
                             circle1.x, circle1.y, circle1.radius) or
        CheckCircularCollision(player1.x, player1.y, player1.radius,
                             circle2.x, circle2.y, circle2.radius) then
        player1.collided = true
    else
        player1.collided = false
    end
end
```

```
function love.draw()  
  love.graphics.setColor(255,255,255)  
  love.graphics.circle("fill",circle1.x, circle1.y, circle1.radius, 100)  
  love.graphics.circle("fill",circle2.x, circle2.y, circle2.radius, 100)  
  
  if player1.collided == true then  
    love.graphics.setColor(255,0,0)  
  end  
  
  love.graphics.circle("fill",player1.x, player1.y, player1.radius, 100)  
end
```



Example 1: Collision Detection with Physics



Example 1: Collision Detection with Physics

```
require "vector2"
```

```
main.lua
```

```
local position = vector2.new(400, 100)
local velocity = vector2.new(0, 0)
local playersize = vector2.new(30, 60)
local maxspeed = 400
local mass = 1
local frictioncoefficient = 300
local onGround = false
```

```
local groundposition = vector2.new(50, 550)
local groundsize = vector2.new(700, 50)
```

```
local box1position = vector2.new(500, 450)
local box1size = vector2.new(50, 50)
```

```
function love.update(dt)
    local acceleration = vector2.new(0, 0)
    local gravity = vector2.new(0, 500)
    acceleration = vector2.applyForce(gravity, mass, acceleration)
    ...
end
```

Example 1: Collision Detection with Physics

main.lua

```
local friction = vector2.mult(velocity, -1)
friction = vector2.normalize(friction)
friction = vector2.mult(friction, frictioncoefficient)
acceleration = vector2.applyForce(friction, mass, acceleration)

local movedirection = vector2.new(0, -1)
if love.keyboard.isDown("right") then
    local move = vector2.new(500, 0)
    acceleration = vector2.applyForce(move, mass, acceleration)
    movedirection.x = 1
end
if love.keyboard.isDown("left") then
    local move = vector2.new(-500, 0)
    acceleration = vector2.applyForce(move, mass, acceleration)
    movedirection.x = -1
end
if love.keyboard.isDown("up") and (onGround) then
    local jump = vector2.new(0, -50000)
    acceleration = vector2.applyForce(jump, mass, acceleration)
    movedirection.y = 1
    onGround = false
end
end
```

Example 1: Collision Detection with Physics

```
local futurevelocity = vector2.add(velocity,
                                   vector2.mult(acceleration, dt))
futurevelocity = vector2.limit(futurevelocity, maxspeed)
local futureposition = vector2.add(position,
                                   vector2.mult(futurevelocity, dt))
local collisiondir1 = GetBoxCollisionDirection(futureposition.x,
                                               futureposition.y, playersize.x, playersize.y,
                                               groundposition.x, groundposition.y, groundsize.x,
                                               groundsize.y)
if not (collisiondir1.x == 0 and collisiondir1.y == 0) then
    if collisiondir1.y == movedirection.y then --down collision
        velocity.y = 0
        acceleration.y = 0
        onGround = true
    elseif collisiondir1.y == 1 then --up collision
        velocity.y = 0
        acceleration.y = 0
    elseif movedirection.x ~= collisiondir1.x then --side collision
        velocity.x = 0
        acceleration.x = 0
    end
end
end
```

Example 1: Collision Detection with Physics

```
local collisiondir2 = GetBoxCollisionDirection(futureposition.x,
                                             futureposition.y, playersize.x, playersize.y,
                                             box1position.x, box1position.y, box1size.x,
                                             box1size.y)
if not (collisiondir2.x == 0 and collisiondir2.y == 0) then
    if collisiondir2.y == movedirection.y then --ground collision
        velocity.y = 0
        acceleration.y = 0
        onGround = true
    elseif collisiondir2.y == 1 then --up collision
        velocity.y = 0
        acceleration.y = 0
    elseif movedirection.x ~= collisiondir2.x then --side collision
        velocity.x = 0
        acceleration.x = 0
    end
end
end

velocity = vector2.add(velocity, vector2.mult(acceleration, dt))
velocity = vector2.limit(velocity, maxspeed)
position = vector2.add(position, vector2.mult(velocity, dt))
end
```


Exercise 3

- 3) Using the code of the collision detection with physics example, implement a scene with at least four platforms (like the one illustrated below).
- Don't worry about code repetitions or optimizations in this exercise.
 - In the next lecture we will see how to store the level geometry in an array and optimize the collision detection process.

